CLAS 12 copies
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BROOKLYN COLLEGE

FINAL EXAMINATION— FALL 2005
MATHEMATICS 37.1 SECTION TF9

SHOW ALL WORK.

1 *a*) Evaluate $\cos(2x+y^2)$ for $x=2\pm 0.03$ and $y=1\pm 0.07$. (The arguments of cosine are radians, and not degrees.)

b) Assume that the differentiable function f has n zeros in the interval (a,b). Explain why f' has at least $n-1$ zeros in the same interval.

2 *a*) Estimate the error of Lagrange interpolation when interpolating $f(x)=\ln x$ at $x=5$, using the interpolation points $x_1=2$, $x_2=3$, and $x_3=6$.

b) Define the sequence $\{x_n\}_{n=1}^{\infty}$ by $x_1=1$ and

$$x_{n+1}=\frac{\frac{2}{x_n}+x_n}{2}.$$

Explain why x_n converges to $\sqrt{2}$.

3 *a*) We want to evaluate

$$\int_2^3 \frac{dx}{\ln x}$$

using the composite trapezoidal rule with five decimal precision, i.e., with an error not exceeding $5\cdot 10^{-6}$. What value of n should one use when dividing the interval $[2,3]$ into n parts?

b) Assume $f(\alpha)=\alpha$, and, for some $c>0$, we have $|f'(x)|<q$ for every x in the interval $(\alpha-c,\alpha+c)$. Show that, we have

$$|f(x)-\alpha|<q|x-\alpha|$$

for any x the above interval. (Hint: use the Mean-Value Theorem).

4 *a*) Given the differential equation $y'=-y^2$ with the initial condition $y(0)=1$, do one step of the Taylor series method to calculate $y(.1)$. For this, use the first four terms of the Taylor series (that is, the last term to be used will involve the third derivative of y).

b) In estimating the error of the trapezoidal rule and Simpson's rule, the following lemma was used.

Lemma. *Let $g(x)\ge 0$ for all $x\in[a,b]$. Assume f is differentiable on (a,b), and for each $x\in[a,b]$ we have $\xi_x\in(a,b)$. Then there is an $\eta\in(a,b)$ such that*

$$\int_a^b f'(\xi_x)g(x)\,dx=f'(\eta)\int_a^b g(x)\,dx,$$

provided that the integrals on both sides of this equation exist.

Briefly outline the proof of this lemma.

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(continued)

5 *a*) Given

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 6 & 4 \\ 0 & 4 & 8 \\ 0 & 0 & 3 \end{pmatrix},$$

write A as $L'U'$ such that L' is a lower triangular matrix and U' is an upper triangular matrix such that the elements in the main diagonal of U' are all 1's.

b) Describe what an orthogonal matrix is.

6 *a*) Solve the equation

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ 14 \end{pmatrix}.$$

b) Find the characteristic polynomial of the matrix

$$A = \begin{pmatrix} 2 & 2 \\ 6 & 3 \end{pmatrix}.$$

c) Find the eigenvalues of the matrix given in Part *b*).