Department of Mathematics
Brooklyn College

Final Examination—Fall 2001
Mathematics 4.3

INSTRUCTIONS: *Answer ALL problems of Part I and any FOUR problems of Part II. Show all work.*

PART I: *Answer ALL THREE questions.*

1. Evaluate the following:

   (a) $\displaystyle\int \frac{5x-1}{(x+1)(x-1)^2}\,dx$ (b) $\displaystyle\int_0^{\pi/2} x^2\cos(2x)\,dx$

   (c) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$

2. Determine the convergence or divergence of each of the following series. For each convergent series, specify if the convergence is absolute or conditional. *Fully justify your conclusion for each example.*

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{(3n-2)(2n+3)}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2n}{3n+1}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^3}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$

3. Consider the region R bounded by the line $y = 5x$ and by the parabola $y = x^2 + x$. Set up, but *do not evaluate*, the definite integrals for each of the following. Always integrate along the x axis.

   (a) To find the area of R.

   (b) To find the volume of the solid that results when R is rotated about the x axis.

   (c) To find the volume of the solid that results when R is rotated about the y axis.

   (d) To find the length of the portion of the parabola bounding R.

(continued)

PART II: *Work any FOUR of FIVE problems.*

4. (a) Find the interval of convergence of the power series $f(x) = \sum_{n=1}^{\infty} \frac{(x+3)^n}{n4^n}$.

   For each endpoint of the interval of convergence, specify whether you have absolute convergence, conditional convergence, or divergence.

   (b) Given the series in Part (a), find $f(-3.25) = f\left(-3 - \frac{1}{4}\right)$ with three decimal precision (i.e., with an error less than or equal to 0.0005). Show your work. *Do not add more terms of the series than necessary* to obtain your answer.

5. (a) Find the Maclaurin series for $f(x) = e^{-x^2}$, given the Maclaurin series, convergent on the whole real line, for e^x:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots .$$

   (b) Using your result in Part (a), find the Maclaurin series for $\int_0^x e^{-t^2}\, dt$.

6. (a) Evaluate $\lim_{x \to 0} (1 - 2x)^{3/x}$.

   (b) Given $f(x) = e^{\arctan x} + \ln(\arcsin x)$, find $f'(x)$.

7. (a) Set up, but *do not evaluate*, the integral describing the area of the region R that is inside the curve $r = 3\sin\theta$ and outside the curve $r = 1 + \sin\theta$. Carefully sketch the region R.

   (b) Evaluate $\sum_{n=2}^{\infty} 3^{n-1} 2^{5-2n}$.

8. (a) (i) Explain why the integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ is improper. (ii) Decide whether or not this integral is convergent. (iii) If it is convergent, evaluate the integral.

   (b) Find the length of the arc of the curve $r = e^{3\theta}$ (given in polar coordinates) for $0 \le \theta \le \pi$.

PLEASE INDICATE, ON THE FRONT COVER OF YOUR EXAMINATION BOOK, THE NUMBER OF THE PROBLEM YOU ARE OMITTING IN PART II.