Department of Mathematics - Brooklyn College
Mathematics 4.3 - Final Examination - Spring, 2003

Name _____ Section _____

Part I: Do (**all**) problems in this part. (48 points)

1. Evaluate the following:

(a) $\int xe^{3x+2}\,dx$

(b) $\int \frac{3x+1}{x(x-1)^2}\,dx$

(c) $\displaystyle\int \frac{3x}{x^2 - 4x + 5}\, dx$

2. Determine whether the following series converge or diverge. Fully justify your conclusion.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3 + 5n}{3n^5 + 7}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{e^{n+2}}{7^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$

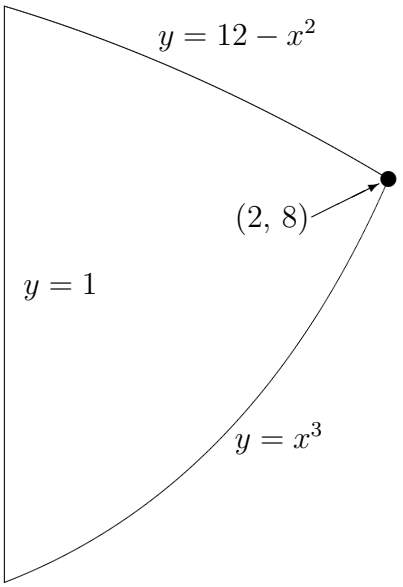
(d) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

3. Consider the region R pictured. Set up, but do **not** evaluate, the definite integrals for each of the following. Always integrate along the x-axis.
   (a) the area of R.

   (b) the volume of the solid generated by revolving R about the x-axis.

   (c) the volume of the solid generated by revolving R about the y-axis.

   (d) the length of the part of the curve $y = 12 - x^2$ that bounds R.



Part II. Do any **four** problems. (13 points each)

4. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n}\,5^n}$. Don't forget to check the endpoints.

5. The MacLaurin series for $\sin x$ is $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

   (a) Find the MacLaurin series for $\sin 3x$.

   (b) Find the MacLaurin series for $\int_0^x \sin(t^2)\,dt$

   (c) Use the MacLaurin series for $\sin x$ to approximate $\sin \frac{1}{2}$ with four decimal precision (i.e., with an error less than .00005). Explain how you determined how many terms of the series to use.

6. (a) Find the terms through degree 3 of the Taylor series for $f(x) = \sqrt{7 - 3x}$ about $x = 2$.

   (b) Evaluate $\lim_{x \to 0} (5x^2 - 2\sin x + 1)^{2/x}$.

7. (a) Use Simpson's Rule and the information given in the table below to approximate $\int_3^6 f(x)\,dx$.

x	3	3.5	4	4.5	5	5.5	6
f(x)	1.8	2.1	2.4	3.0	2.4	2.0	2.6

   (b) Determine if the improper integral $\int_3^\infty \frac{x}{(x^2-1)^2}\,dx$ converges or diverges. If it converges, find its value.

8. (a) Set up, but do not evaluate, an integral that represents the area of the region R inside $r = 8\sin\theta$ and outside $r = 3 + 2\sin\theta$. Carefully sketch the region R.

   (b) Find $f'(x)$ if $f(x) = \arcsin 3x + \ln(\arctan x)$.