Department of Mathematics
Brooklyn College

Final Examination—Fall 2005
Mathematics 4.3

PART I. *Do* **all** *problems in this part.*

1. Evaluate the following:

   (a) $\displaystyle\int \frac{2x^2 - 11x + 5}{(x+1)(x-2)^2}\,dx$ (b) $\displaystyle\int (5-x)e^{3x}\,dx$

   (c) $\displaystyle\int \frac{\sqrt{x^2-4}}{x^4}\,dx$

2. Determine if the following series converge or diverge. Fully justify your conclusions.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+5}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{4n^2-5}{3n^3+2n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{n+3}}{\sqrt{n}\cdot 4^n}$

3. Consider the region R pictured to the right. Set up (**do not evaluate**) definite integrals for each of the following. Always integrate along the x-axis.

   (a) the area of R,

   (b) the volume of the solid that results when R is rotated about the x-axis,

   (c) the volume of the solid that results when R is rotated about the y-axis,

   (d) the length of the portion of the parabola that bounds R.

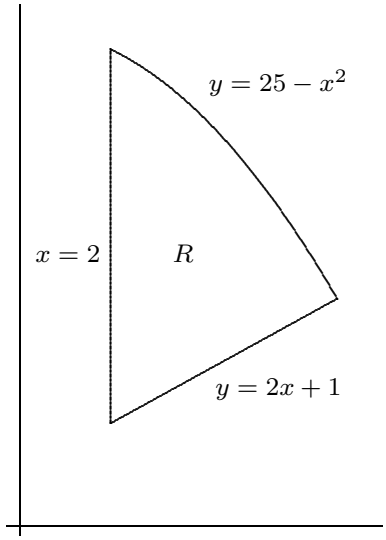

Figure not drawn to scale.

All computer processing for this document was done under Red Hat Linux. $\mathcal{AMS}$-TEX was used for typesetting. PICTEX was used for the diagram, together with the programming language Perl for generating the data points.

(continued)

PART II. *Do any* **four** *of the five problems in this part.*

4. Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(2x-3)^n}{5^n}$. Remember to check the endpoints. If the series is convergent at an endpoint of the interval of convergence, indicate whether the convergence is absolute or conditional.

5. (a) Use Simpson's Rule with $n = 4$ to approximate $\displaystyle\int_0^2 \frac{1}{1+x^3}\,dx$. Before doing any calculations, write out the expression you are trying to evaluate. Round your answer to four decimal places.

   (b) Evaluate $\lim_{x\to 0}(1+5x)^{\cot 2x}$.

6. (a) Determine if the improper integral $\displaystyle\int_2^{\infty} \frac{x}{(x^2-1)^{3/2}}\,dx$ is convergent or divergent. If it is convergent, find the value of the integral.

   (b) Find $f'(x)$ if $f(x) = \arctan 5x + \ln(\arcsin 2x)$.

7. (a) Set up (**do not evaluate**) a definite integral that represents the area of the region R inside $r = 3\cos\theta$ and outside $r = \sqrt{3} + \cos\theta$. Carefully sketch the region R.

   (b) Find the Taylor polynomial of degree three for $f(x) = \dfrac{1}{(x-3)^2}$ about $x = 4$.

8. (a) Use the Maclaurin series $\displaystyle\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ to find the Maclaurin series for

   (i) $x\sin(3x)$

   (ii) $\displaystyle\int_0^x \sin(t^2)\,dt$

   (b) Express $\sin\dfrac{1}{2}$ as an infinite series and use that series to approximate $\sin\dfrac{1}{2}$ to an accuracy of four decimal places. *Show all work.*

END OF EXAMINATION