Department of Mathematics
Brooklyn College

Final Examination—Spring 2005
Mathematics 4.3

PART I. *Do* **all** *problems in this part (48 points).*

1. Evaluate the following:

   (a) $\displaystyle\int \sqrt{x}\ln x\,dx$ (b) $\displaystyle\int \frac{3x^2+x-5}{(x-3)(x+2)^2}\,dx$

   (c) $\displaystyle\int \frac{(x-3)^3}{\sqrt{x^2-6x}}\,dx$

2. Determine whether the following series converge or diverge. Fully justify your conclusions.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{n+4}}{5^n}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\,\frac{n}{5n+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{3n+1}{(n+1)(2n-1)}$

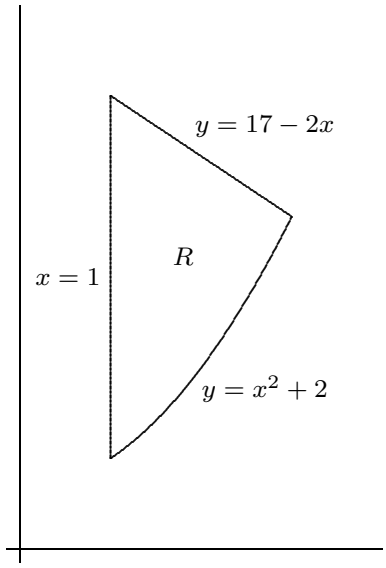
3. Consider the region R shown in the figure. Write an integral formula to find each of the following (DO NOT EVALUATE):

   (a) the area of R,

   (b) the volume of the solid that results when R is rotated about the x-axis,

   (c) the volume of the solid that results when R is rotated about the y-axis,

   (d) the length of the part of the curve $y = x^2+2$ that bounds R.



All computer processing for this document was done under Red Hat Linux. $\mathcal{AMS}$-TEX was used for typesetting. PICTEX was used for the diagram, together with the programming language Perl for generating the data points.

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PART II. *Do* **any four** *out of five problems in this part (52 points).*

4. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{\sqrt{n+1}}$. If the series is convergent at an endpoint of the interval of convergence, indicate whether the convergence is conditional or absolute.

5. (a) Use Simpson's use with $n = 4$ to approximate $\int_0^4 \sqrt{1+x^3}\,dx$. Before doing any calculation, write out the expression you are going to evaluate. Round your answer to four decimal places.

   (b) If $f(x) = \arctan(2x+1) + \ln(\arcsin 5x)$, find $f'(x)$.

6. (a) Determine if the improper integral $\int_0^{1/2} \frac{1}{\sqrt{1-4x^2}}\,dx$ converges or diverges. If it converges, evaluate the integral.

   (b) Evaluate $\lim_{x\to 1}(2x^2-1)^{\frac{5}{x-1}}$.

7. (a) Set up, but do not evaluate, an integral that represents the area of the region R that is inside $r = 5\cos\theta$ and outside $r = 2 + \cos\theta$. Carefully sketch the region R.

   (b) Find the expansion through the 3rd degree term of the Taylor series for $f(x) = 1/\sqrt{2x-3}$ about $x = 2$.

8. Given the Maclaurin series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$,

   (a) find the Maclaurin series for $\int_0^x e^{-t^2}\,dt$,

   (b) express $e^{-1/2}$ as an infinite series and determine how many terms of your series are needed to approximate $e^{-1/2}$ to an accuracy of three decimal places. *Show all work.*

END OF EXAMINATION