CENTRIPETAL FORCE

Purpose

- **a.** To study the characteristics of uniform circular motion.
- **b.** To experimentally measure centripetal force in a circular motion.

Theory

When an object of mass M is revolving in a circular motion of radius R, the object is in accelerating motion. The radial component of the acceleration, called centripetal acceleration is given by

$$a_c = \frac{v^2}{R},\tag{1}$$

which is directed to the center of the circular orbit. In a uniform circular motion, the speed, v, of the velocity vector is constant. Only direction is changing and the velocity is tangential to the orbit. So, the net force, called centripetal force, is also directed towards the center and given by

$$F_c = Ma_c = \frac{Mv^2}{R},$$
(2)



Figure 1. A uniform circular motion.

A centripetal force is not an extra force that occurs by itself. It is the resultant of some other forces such as tension, gravity, friction, elasticity, electric attraction etc. that cause the object to move in a circular path.

According to the Equation (2), centripetal force is proportional to the square of the speed for an object of given mass M rotating in a given radius R. You are going to experimentally verify this relationship in this lab. Similarly, you can investigate relation between any two quantities experimentally by keep two other quantities constant.

Since the motion is uniform, the speed can be determined by measuring the time for revolution by using the following relations

$$v = R\omega = R2\pi f = \frac{2\pi R}{T},\tag{3}$$

where ω , *f* and *T* are angular speed, frequency, and period of revolution respectively. In terms of *f* and *T*, we can rewrite the Equation (2) as follows

$$F_{c} = 4\pi^{2} M R f^{2} = \frac{4\pi^{2} M R}{T^{2}},$$
(4)

By measuring the time of revolution of a uniform circular motion, centripetal force can be determined.

Apparatus

Centripetal force apparatus, digital stopwatch, set of weights: 100 x 10, 50, 20 x 2, 10 g, ruler, balance.

Description of apparatus

In this lab a metal bob is rotated in a uniform circular motion. Experimental set up for this lab is shown in Figure 2. The apparatus consists of a vertical shaft supported by bearing system on a horizontal base. The bearing system allows the shaft to rotate with minimum friction. On top of the shaft, it has a

horizontal sliding arm. A metal bob is hung by a string on one side of this arm and a counter weight is attached on the other side. The metal bob has a pointed conical shape at the bottom. The bob is attached to the shaft by a spring. Tension in the spring can be varied by hooking the spring on to different holes on a metal strip connected to the shaft. There is a movable long vertical pointer attached to the base to find the radius of rotating bob. The pointer has a resilient tine (a flat spring) on the upper end. You can rotate the bob by spinning the knurling located near the bottom of the shaft. In order to maintain uniform circular motion, you should twirl the shaft just enough so that the tip of the cone at the bottom of the metal bob lines up and touches the tine on the pointer. With a little practice, you should be able to maintain uniform circular motion.



Figure 2. Centripetal force apparatus.

Figure 3a shows the free body diagram for the rotating bob in uniform circular motion. The weight of the mass is balanced by the tension in the suspending string. The centripetal force is provided by the tension in the spring attaching the bob to the shaft. We can measure the tension in the spring in a static state, i.e., without rotation as shown in the Figure 3b. When the bob is not rotating, it will be pulled toward the shaft. A force can be applied in opposite direction to the tension in the spring. A string is connected to the other side of the bob, pass over a pulley and a load is hung at the other end of the string as shown in Figure 2. The mass on the load is adjusted so that the pointer and the cone again line up. At this condition, the value of the force exerted by the string is equal to the spring force which is providing the centripetal force.

$$F_{sp} = mg$$
,

where *m* is the total mass of the load including the hanger. This force is equal to the centripetal force for holding the bob at the same radius when it is rotating.



Figure 3. Free-body diagrams for (a) dynamic and (b) static measurements.

Procedure

Before you start the experiment, if the base is not leveled, adjust the thumb screws in the base to level it. You can also adjust the position of the arm and counter weight. What do you think is the role of counter weight in this lab?

Part 1. Dependence of centripetal force (F_c) on the speed of rotation (v) at constant radius

In this part of the experiment, you are going to investigate the relation between the speed and centripetal force of an object rotating in a uniform circular motion for a given mass and radius of the orbit. What do you expect the relation between them based on the explanation given in the theory section?

- 1. Release the spring and measure the mass M of the bob. Now hang the bob on the arm. Do not connect the spring yet. Now, adjust the position of the arm, so that the cone of the bob is lined up with the pointer. Measure the radius of rotation, R, which is the distance between the center of the shaft and the pointer, and record it in Table 1.
- 2. Hook the spring to one of the hole on the metal strip and the bob. Try to spin the shaft. **Watch out** that your head does not get hit by the rotating parts. The rotation should be just fast enough that tip of the cone of the bob hits the resilient tine on the pointer to produce a sound. You should keep the rotation steady so that the cone hitting the tine gives a regular sound. Practice for a while. While rotating, if the whole apparatus imbalances, you may need to adjust the position of the counter weight. Once you are comfortable spinning uniformly, using a digital stopwatch measure the time, *t*, for 20 revolutions and record it.
- 3. Stop the rotation. In order to measure the centripetal force for the circular motion, attach a string to the other side of the bob and run the string over the pulley to a weight hanger. Adjust the slotted mass on the hanger so that the pointer and the cone line up again. Record the total mass *m*, i.e., including the mass of the hanger in your data sheet.
- 4. In order to change the centripetal force, hook the spring end to a different hole on the metal strip. Repeat steps 2 and 3.
- 5. Repeat these steps for all the holes on the metal strip.

Part 2. Dependence of the period of rotation (T) on the mass (M) at constant radius

In this part of the experiment, you are going to vary the mass of the rotating object and see how the periods of rotations change by keeping the radius and centripetal force constant. Remember the period of the rotation is related to the speed of the object in a uniform circular motion. What do you expect the relation between them based on the explanation given in the theory section?

- 1. Keep the same set up as in Part 1. Note down the mass of the bob and the radius in Table 2. Connect the spring to one of the holes on the metal strip.
- 2. Rotate the shaft uniformly as before (You should hear a regular sound from the cone of the mass hitting the pointer.) Measure the time, t, for 20 revolutions using the stopwatch.
- 3. Stop the rotation. In order to determine the centripetal force, attach the string of the hanger to the bob as in part 1. Adjust the slotted mass on the hanger to line up the pointer to the cone. Record your measurements in the data sheet.
- 4. Now, add 100 g to the metal bob, tighten it with a nut, and record the total rotating mass, M in Table 2. Repeat step 2 to find the time for 20 revolutions of the rotating bob. You do not have to repeat the step 3 since the radius and the tension of the spring are same.
- 5. Repeat step 4 by adding another 100 g on the metal bob.

Part 3. Dependence of the speed (v) on the radius (R) at a constant centripetal force

In this part of the experiment, you are going to keep the Mass, M and centripetal force constant and see how the speed (v) varies by changing the radius (R) of a uniform circular motion. What do you expect the relation between them based on the explanation given in the theory section?

- 1. Set the pointer at the closest position to the shaft, i.e., at the smallest possible radius. Release the spring from the metal bob and adjust the position of the arm so that the tip of the cone at the bottom of the bob is lined up with the pointer. Measure the radius of rotation, R, as before and record in Table 3.
- 2. Now, connect the spring to the bob and hook on one of the holes on the metal stripe. Determine the centripetal force needed for a uniform circular motion for this radius and mass as before.
- 3. Disconnect the hanger string. Now, spin the shaft uniformly as before and measure the time t for 20 revolutions, and record in the Table 3.
- 4. Remember, you are doing this part of the experiment at constant centripetal force. Connect the spring to the hole on the metal strip closest to the shaft. Then connect the string of the hanger **with the previously measured mass** to the metal bob. Adjust the arm so that the suspending string is vertical and then adjust the pointer in the position to line it up with the cone. Record the radius in the data sheet.
- 5. Disconnect the hanger string. Measure the time, *t*, for 20 revolutions by spinning the shaft uniformly and record your result in Table 3.
- 6. Now change the position of the hole on the metal stripe to vary the radius of the circular path. Repeat the steps 4 and 5 for all holes on the metal strip.

Computation and Analysis

From each measurement of the time for 20 revolutions, calculate the time period *T*. Calculate F_{sp} for each static measurement from equation (5). Using equations (2) and (4) calculate the theoretical value of the centripetal force, F_c for Table 1 compare it to static measurements F_{sp} .

From the data in Table 1, plot graphs of F_{sp} vs. v^2 . Find the slope in the graph. What do you get from the slope of the graph? Calculate the theoretical value of the slope and compare with the result from the graph. From the data in Table 2, plot a graph T^2 vs. M. Find the slope from the graph. Calculate the theoretical value of the slope and compare it to the slope from the graph. From the data in Table 3, plot the graph v^2 vs. R. Find the slope from the graph. Calculate the theoretical value of the slope from the graph. Calculate the theoretical value of the slope from the graph. Calculate the theoretical value of the slope and compare it to the slope from the graph.

Questions

- 1. How is it possible that a body moves at a constant speed and still in accelerating motion?
- 2. When a car is going around a circular track with constant speed, what provides the centripetal force necessary for circular motion?
- 3. What are directions of acceleration and net force if the speed of an object is changing while rotating in a circular motion?
- 4. In this experiment, what would be the effect if the point on the arm hanging the bob and the pointer are not on the same vertical line in the experiment?
- 5. In this experiment, if there is no spring attached and the bob is rotated at a constant speed, what provides the centripetal force? Draw a diagram to explain your answer.

Data Sheet

Date experiment performed:

Name of the group members:

Part 1. Dependence of centripetal force on the speed of rotation (v) at constant radius

Mass of the metal bob (M) =

Radius (R) =

Table 1.

Trial	Time for 20 rev, t (sec)	Period, T (sec)	Speed, v (m/s)	v^2 $(m/s)^2$	Hanging mass, m (kg)	Spring force, <i>F_{sp}</i> (N)	Centripetal force, F_c (N)	% difference

Slope from the graph of F_{sp} versus $v^2 =$

Theoretical value of the slope =

% difference =

Part 2. Dependence of the period of rotation (T) on the mass at constant radius

Radius (R) =

Hanging mass (m) =

Spring force $(F_{sp}) =$

Table 2.

Trial	Total mass on the bob, <i>M</i> (kg)	Time for 20 rev, t (sec)	Period, T (sec)	T^2 (sec^2)	

Slope from the graph T^2 versus M =

Theoretical value of the slope =

% difference =

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Part 3. Dependence of the speed (v) on the radius (R) at a constant centripetal force

Mass of the bob (M) =

Total hanging mass (m) =

Spring force $(F_{sp}) =$

Table 3.

Trial	Time for 20 rev, t (sec)	Period, T (sec)	Radius, <i>R</i> (m)	Speed, v (m/s)	$\frac{v^2}{(m/s)^2}$	Centripetal force, F_c (N)

Average $F_c =$

Slope from the graph of v^2 versus R =

Theoretical value of the slope =

% difference =