

CONSERVATIVE FORCE SYSTEMS

Purpose

- To investigate Hooke's law and determine the spring constant.
- To study the nature of conservative force systems using a spring-mass system as an example.

Theory

I. Hooke's law and Spring constant

When an object of mass m is attached at the lower end of a vertical spring, it elongates and comes to equilibrium. From Hooke's law, the spring force $F = -k\Delta X$, where ΔX is the displacement (**elongation**) of the spring from its **unstretched** position as shown in the Figure 1, and k is the spring constant. The minus sign shows that the force F acts in such a direction as to reduce the magnitude of the displacement. At the equilibrium position the spring force is balanced by the weight of the object attached. We are assuming an ideal (non-dissipative) spring with negligible mass. Thus

$$k\Delta X = mg. \quad (1)$$

In the first part of this lab, we will investigate this relation and determine the spring constant for a spring.

II. Conservation of Energy

In a conservative force system the work done by the force can be expressed as the negative of the change in the potential energy ($W_c = -\Delta U$). Potential energy decreases (increases) when a conservative force does positive (negative) work. Thus the total mechanical energy (kinetic plus potential energy) is always conserved.

We can calculate the kinetic and potential energies by **measuring** the **velocities** and **positions** of a mass attached to the spring using a motion detector.

- What other information do you need to calculate the kinetic energy?
- What other information do you need to calculate the spring potential energy?
- What other information do you need to calculate the gravitational potential energy?

When the hanging mass is stretched down and released, it will oscillate about the equilibrium position. The kinetic energy is given by

$$KE = \frac{1}{2}mv^2. \quad (2)$$

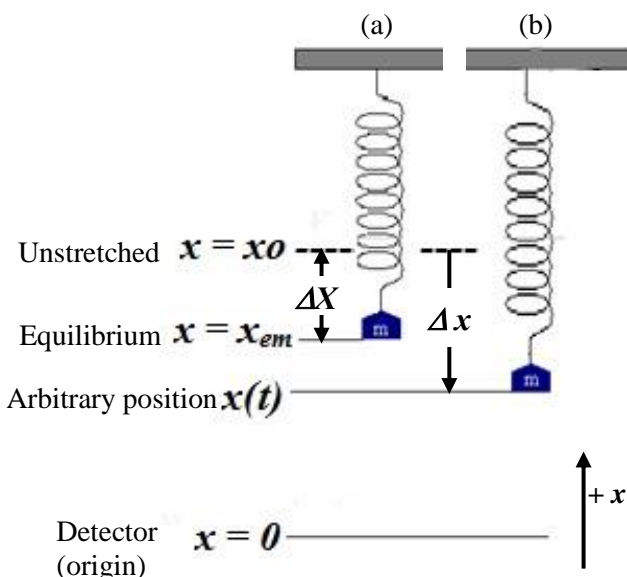


Figure 1. Spring-mass system in (a) equilibrium and (b) oscillating. x_0 , x_{em} and $x(t)$ are positions of the bottom of the hanger.

Since the hanging mass is oscillating vertically, it involves both the gravitational and the spring potential energies. The spring potential energy, u_s , is given by

$$u_s = \frac{1}{2} k(\Delta x)^2, \quad (3)$$

where Δx is the displacement (**elongation**) of the lower end of the **spring relative to its unstretched position** x_0 at any time t and is measured positively upward (see Figure 1). Note that the spring potential energy (u_s) is zero when it is unstretched.

Now, assuming the gravitational potential energy is zero when $x = x_{em}$ (We can make this assumption since it is only differences in potential energy that will be physically significant.), at arbitrary x is given by

$$u_g = mg(x-x_{em}). \quad (4)$$

See Figure 1a.

In our experiment x at any time t will be measured positively upward **from a motion detector to the bottom of a hanger**. Thus, the total mechanical energy is

$$\text{Total Energy} = \frac{1}{2} mv^2 + mg(x-x_{em}) + \frac{1}{2} k(\Delta x)^2. \quad (5)$$

The spring force and the gravitational force are conservative forces. If there are no other forces acting on our system then, from the principle of conservation of energy, the total energy is conserved; i.e., the total energy does not change. In part II of this lab we will investigate this experimentally.

One can show that the total energy is equal to

$$\frac{1}{2} kA^2 + \frac{1}{2} k(\Delta X)^2 + mgc = \text{constant} \quad (5.1)$$

where A is the amplitude of the oscillation, ΔX is the elongation of the **spring** at equilibrium, and c is the distance from the bottom of the hanger to the c.m. of the weights and hanger.

To take into account the mass of the spring, heretofore ignored, replace m in $\frac{1}{2} mv^2$ in Eqs. (2) and (5) by an effective mass $m_{eff} = m + \frac{1}{3} m_{sp}$, (6)
where m_{sp} is the mass of the spring.

Apparatus

Jolly balance, scale, spring, set of slotted weights (50 and 100 grams), 50 gram slotted mass hanger, rulers, graph paper, motion detector, Vernier data acquisition system, Logger Pro software.

Description of Apparatus

The Jolly balance was invented by the German physicist Philipp von Jolly in 1864. This and the other apparatus that will be used in this laboratory are shown in Figure 2. It consists of a movable arm at the top of a stand pipe. The movable arm has an engraved scale and can be moved up or down by rotating the knurled wheel at the bottom of the balance. A Vernier scale is also attached to increase the resolution of the measurement. A spring is fastened at the top of the movable arm and a weight hanger is hung at the lower end of the spring. It has a movable pointer attached on the stand to mark the position of the spring. It also has a pan on which to put the motion detector.

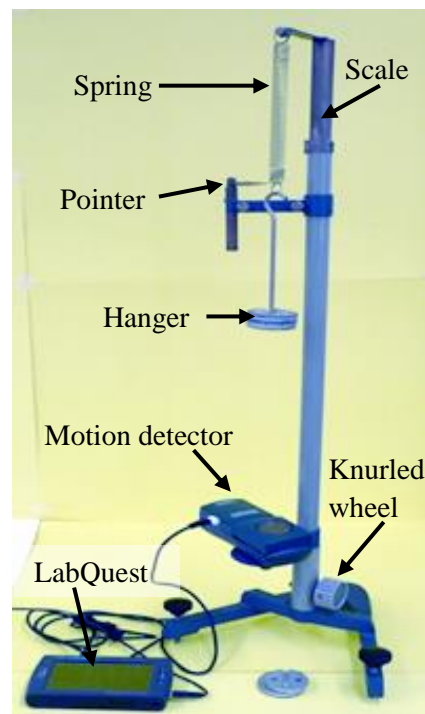


Figure 2. Jolly balance and accessories.

We will use a Vernier motion detector along with a LabQuest data acquisition device to determine the position and velocity of the hanger with respect to time while oscillating. From the data of position and velocity we will plot the graphs of kinetic and potential energies to investigate the conservation principle.

Procedure

Part I. Setting up the apparatus and determining the spring constant (k)

1. Set the scale of the Jolly balance to zero position by adjusting the knurled wheel. Hang the spring on its movable arm if it is not already there. Adjust the pointer tip of the balance to the lowest point of the spring and lock in the position of the pointer.
2. Find the mass of the hanger and place the hanger on the end of the spring. Because of the load, the spring elongates. In order to measure how much it is elongated, raise the Jolly balance using the knurled wheel until the lowest edge of the spring comes back to the tip of the pointer. Read the scale of the Jolly balance to one-tenth of a millimeter using the Vernier scale and record it (ΔX) on the data sheet at the end of the write-up.
3. Now you are going to repeat the previous step by adding masses on the hanger. Suggested total masses, m , (including the hanger) are 100, 150, 200, 250, 300, 350, and 400 grams. Record the scale readings (ΔX) on the data sheet.
4. Before continuing with Part II below, go to the **Computations** section and determine the spring constant, k .

Part II. Measuring position and velocity

You are going to use a motion detector to collect the data of the position and velocity of the hanging mass while oscillating. The motion detector should be connected to the LabQuest interface device and then to the computer to collect data. **Note that the motion detector collects data positively away from the detector.**

1. Place 350 grams on the hanger, so that m is 400 grams. Find and record the mass of the spring, m_{sp} , and the effective mass, m_{eff} .
2. Put the motion detector on the pan of the Jolly balance and adjust the position of the stand so that the bottom of the hanger is about 50 cm directly above the motion detector. The beam of the motion detector should be directed upward to the hanger (see Figure 2).
3. Open Logger Pro in your computer. You have two empty graphs of position versus time and velocity versus time, and a table. Note the equilibrium position, x_{em} , of the bottom of the hanger with the hanging mass from the motion detector and record (on the data sheet) its value shown in the lower left-hand corner.
4. Now, raise or lower the hanger about 5-10 cm and release it gently. Let it oscillate a few times so that the hanging mass oscillates vertically without much side-to-side motion. If it is jiggling, try it again. (Too much compression or extension or pushing it away from the vertical may cause jiggling.) If the mass is oscillating nicely, **collect data for position and velocity** for a few cycles and **save** the data and graphs for further analysis. Look at the graphs carefully. Print the graphs and table. (This is your data.)

How does the velocity change with respect to time?

From the velocity versus time graph, can you tell how the acceleration changes with time?

Locate the positions of the bottom of the hanger where the velocities are **zeroes** and **maxima** (in magnitude). Record them on the data sheet.

From the position graph find the amplitude, A , of the oscillation. Record it.

Determine the time period T of the oscillation from position versus time graph by fitting the data with a sine function thereby obtaining $\omega = 2\pi/T$. Record your values of ω and T .

The spring constant, mass, and period of oscillation, T , are related by means of the equation

$$T = 2\pi \sqrt{\frac{m_{eff}}{k}} \quad (7)$$

Find the spring constant k from Eq. (7) above. Record it.

Calculate the elongation $\Delta X = \frac{mg}{k}$. Record it.

5. From the diagram (Figure 1) you should be able to see (**with some thought**) that Δx , the elongation of the spring at any time t , is given by

$$\Delta x = x - (x_{em} + \Delta X) \quad (8)$$

where x is the distance from the motion detector (i.e., from $x = 0$). (*Hint*: Note that **in this diagram** Δx is negative, and we take ΔX to be a positive number.)

Part III. Calculating and plotting kinetic energy and potential energies versus time

You can calculate kinetic energy, spring potential and gravitational potential energies from the data you collected from the motion detector using the equations (2, 3, 4, and 8).

You have to plot the graphs of kinetic, spring potential and gravitational potential energies versus time for further analysis. Include the graphs in your report.

You can copy your data and use Excel for calculating energies and plotting the graphs for further analysis. OR better yet follow the instructions given below.

Print the graphs in 3, 5, 6, 7, and 8 below after saving them.

a. Plotting Kinetic Energy versus time graph

1. On the computer in Logger Pro window, click '**New Calculated Column**' under '**Data**' menu. Name it 'Kinetic energy' (KE for short name) and 'Joules' in the units box.
2. Under the 'expression', type the right hand side of the formula for kinetic energy (Eq. 2) as in any computer language. (*Hint*: Use * for the multiplication sign and ^ for power.) **Use m_{eff} for the mass.** You may directly use the value for m_{eff} or use it as a parameter and then substitute the value. For v , click 'Variables (Columns)' and choose (velocity). Do not forget to square the velocity in the expression! Once it is done, this will add a column for you with calculated kinetic energy values corresponding to the time column.
3. Now 'Insert' a new **graph** to display the Kinetic Energy (KE) versus time (t) graph and save the graph.

b. Plotting Potential Energy versus time graph

You will be repeating the previous steps to plot graphs for potential energies.

4. Let's do it for the gravitational potential energy first. Add a 'New Calculated Column' under 'Data' menu. Name it 'Gravitational Potential Energy' (GPE for short name) and 'Joules' in the units box. In the expression, this time you are going to use the equation for gravitational potential energy (Eq. 4). For x , click 'Variables (Columns)' and choose (position). This will add a column for you with calculated gravitational potential energy values corresponding to the time column.
5. 'Insert' a new **graph** to display the Gravitational Potential Energy (GPE) versus time (t) graph and save the graph.
6. Similarly, plot a graph for Spring Potential Energy (SPE). Use Eqs. 3 and 8 to write expression.

c. More plots

7. Plot and save a graph of Total Potential Energy (GPE+SPE) versus time.
8. Plot and save a graph Total Energy (KE+GPE+SPE) versus time.

Computations

From your data in Table 1, plot a graph of weight (mg) vs. spring elongation ΔX .

Does the graph suggest a straight line for $mg = k\Delta X$?

Find the slope. **Determine the spring constant.** Record.

Go to Part II.

Calculate the time period from the value of k obtained from Part I. Explain sources of error.

Analyze the graphs. You should find the answers for all the questions given below.

Questions

1. How well does the spring obey Hooke's law?
2. Look carefully at the plots of KE vs t and PE vs t. Locate the positions where the kinetic energy is maximum. Mark them on your graph. What can you say about them?
3. Locate the positions where the kinetic energy is zero. Mark them on your graph. What can you say about them?
4. How does the gravitational potential energy vary in one cycle?
5. How does the spring potential energy vary in one cycle?
6. Compare the graphs of kinetic energy and of total potential energy (GPE+SPE). Explain if there are any interesting features.
7. How does the total energy (kinetic plus potential energies) change with time? (In theory it should be constant. One could show that it is equal to the sum of the KE and the spring and gravitational potential energies at the equilibrium position.)

8. Find your best value of the total energy and record it. Estimate c and record it. (It should be approximately 0.016 m when $m = 0.4$ kg.) Calculate the theoretical value given by Eq. (5.1) and record it. Find and record the percent error.
9. What do you expect the total energy to be after some time?
10. How would the total energy versus time graph look if a non-conservative force, such as air resistance, becomes important?
11. What can you conclude about conservation of total mechanical energy in a spring-mass system?
12. If a large plate were taped to the bottom of the hanger and then the experiment was repeated, predict how the graph of the total energy vs. time would look like.

Data Sheet

Date experiment performed:

Names of the group members:

Part I.

Table 1. Hooke's law and finding Spring constant

m (kg)	mg (N)	ΔX (m)	$mg/\Delta X$ (N/m)
0.050			
0.100			
0.150			
0.200			
0.250			
0.300			
0.350			
0.400			

Spring constant, k from procedure of part I = _____ N/m

Part II. Measuring position and velocity

$m =$ _____ kg

Mass of spring $m_{sp} =$ _____ kg

Effective mass $m_{eff} = m + \frac{1}{3}m_{sp} =$ _____ kg

Position of equilibrium from motion detector, $x_{em} =$ _____ m

Zeros of velocity at $x =$ _____, _____, _____, _____ m

Maxima (magnitude) of velocity at $x =$ _____, _____, _____, _____ m

A (amplitude) = _____ m

$\omega =$ _____ s^{-1}

$T =$ _____ s

k (from Eq. (7)) = _____ N/m

Elongation $\Delta X =$ _____ m

Time period from value of k from part I =

Measured total energy = _____ J

$c =$ _____ m

Theoretical total energy (Eq. 5.1) = _____ J

% difference = _____