EQUILIBRIUM OF COPLANAR FORCES

Purpose

a. To study the composition and equilibrium of coplanar forces.
b. To study the rectangular resolution and equilibrium of coplanar forces.

Theory

Part I: Composition and equilibrium of coplanar forces

The first part of the experiment is to show how several forces whose lines of action lie on a plane and pass through one point, can be balanced by a single force with line of action passing through the same point. The method is to find the one force, called the resultant, which is equal to the sum of the original forces, and then to balance this resultant with an equal and opposite force, called the equilibrant. The resultant of the original forces is found by the method of vector addition.

When a number of forces, $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$, for example, is acting on an object and lines of actions of all the forces pass through one point O as shown in Figure 1a, the resultant of the forces, **R**, can be found by arranging the vectors (forces) in tip-to-tail fashion thereby forming the sides of a polygon as shown in Figure 1b. *The vector that connects the tail of the first vector with the tip of the last one is the resultant of the vectors being added.*

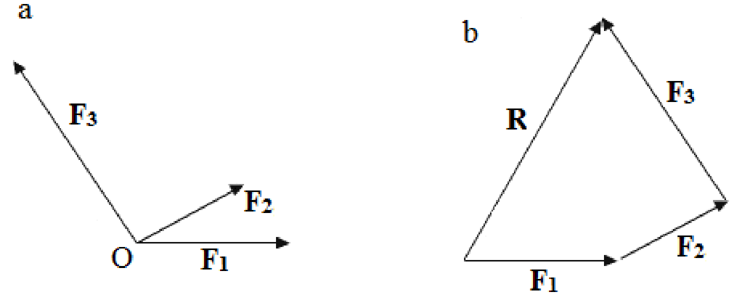

Figure 1: (a) $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$ are forces acting at point O. (b) Finding the resultant force **R** by completing a polygon.

Part II: Rectangular resolution and equilibrium of coplanar forces

The purpose of this part of the experiment is to show how a *single* force may be resolved into two mutually perpendicular *components* which together may be regarded as equivalent to the given force. As shown in Figure 2, a force vector **F** can be resolved along mutually perpendicular x- and y-axes. By drawing perpendicular lines from the tip of the force vector **F** to the axes, the projections F_x and F_y along the x- and y-axes are the rectangular components of the force **F**. The combined effect of F_x along x-axis and F_y along y-axis is the same as that of **F**.

If there is more than one force acting on an object, all the forces can be resolved in rectangular components. *The sum of all the x-components will be the x-component of the resultant force and the sum of the y-components will be the y-component of the resultant.*

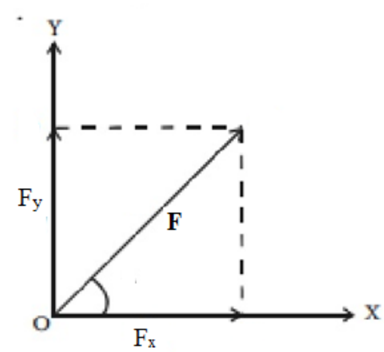

Figure 2: Rectangular resolution of a force, **F**.

Apparatus

Circular table stand, labeled in degrees; ring and pin, for centering; four pulleys; four weight-hangers, each with a mass of 50 grams; four 100 gram masses; four 50 gram masses; two 20 gram masses; two 10 gram masses; one set of small masses (5, 2, 2, 1 gram); protractor; ruler.

Description of apparatus

You will first determine the resultant of the forces by a graphical method and find the force needed to bring them to equilibrium. The diagrams that you make for this experiment should be made with a sharp pencil on plain white paper. Once you have obtained a result for the equilibriant, you will use a force table as shown in Figure 3 to verify your result experimentally. The force table has pulleys that can be moved around the edge of the plate. The plate is marked with angles to change the direction of the force. You can apply the forces on a ring by hanging weights from the strings passing over the pulleys at the edge of the force table. **The pulleys are very brittle; handle them carefully, and avoid dropping them.**

Figure 3: Force table

Procedure

Part I: Composition and equilibrium of coplanar forces

You are to find the *resultant and equilibrant* of three forces whose lines of actions pass through the same point. You should first draw a space diagram, as described below, to serve as a guide in arranging the force table.

1. Choose a point, about 7 cm from the top of your sheet of white paper, for the center of the space diagram, and draw a circle about this point to represent the force table. You may use one of the circular weights for this purpose. Indicate on the circle the 0°, 90°, 180°, and 270° positions. See Figure 4.
2. Draw three lines radiating away from the center: one in the first quadrant to represent a force $\mathbf{F_1}$, one in the second quadrant to represent a force $\mathbf{F_2}$, and one in the third quadrant to represent a force $\mathbf{F_3}$. You may select any convenient direction for the three forces. Choose masses of the hanging weights in the range of 100 grams to 150 grams that exert forces $\mathbf{F_1}$**,** $\mathbf{F_2}$**,** and $\mathbf{F_3}$**.** For now, your force vectors don't have to be drawn to scale. Indicate on your diagram the magnitudes of the three forces and also the angle of each with the 0° - 180° line. *How much is the force in Newton (N) caused by a hundred gram of hanging mass?*

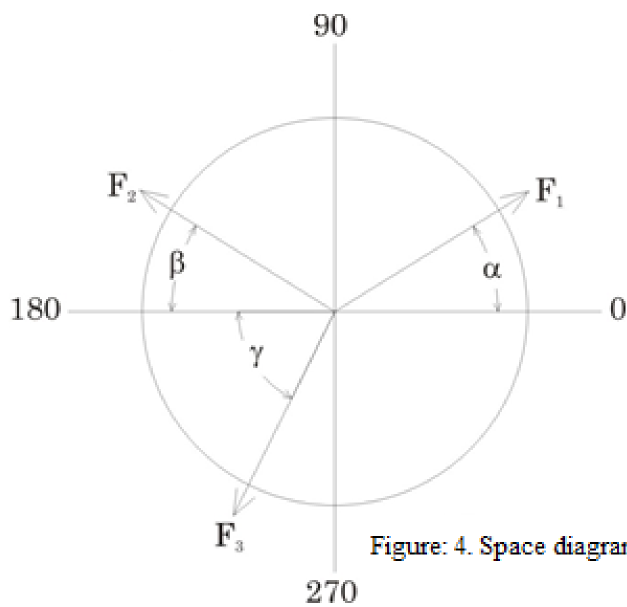

Figure: 4. Space diagram

Now, you are going to construct a force diagram that you can use to determine graphically the resultant and equilibrant of the three forces. **This diagram must be drawn carefully to scale** so that the lines represent the forces accurately. Follow procedure below to draw force diagram (Figure 5).

3. On the lower half of your paper, draw a line to represent $\mathbf{F_1}$. This line should be parallel to the $\mathbf{F_1}$ in the space diagram. Since the length of the vector is proportional to its magnitude, make the length such that 1.0 cm corresponds to the force due to 10 grams. Indicate the direction of the force by an arrow head.

4. Now, using the protractor to mark the angle $\alpha + \beta$, draw $\mathbf{F_2}$, with its tail at the head of $\mathbf{F_1}$, and its head pointing in the same direction as $\mathbf{F_2}$ in the space diagram. Again let 1.0 cm correspond to 10 grams. Similarly, draw $\mathbf{F_3}$. Each force line should be parallel to its direction in Figure 4.

5. Draw the resultant **R** and the equilibrant **E** = **-R**. Measure the length of **R** and the magnitude of θ, the angle between 0° and the equilibrant.

6. **Now check your work experimentally**. Pin the metal ring to the center of the force table top. Clamp pulleys at the positions $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$, and the equilibrant. Pass one string over each pulley and suspend from it a weight hanger and a combination of weights which will yield the forces indicated in your diagram. Remember that the hangers weigh 50 grams each. Adjust the strings so that they point exactly toward the center of the ring where the pin should be located. Otherwise the angles between the strings will not be the angles indicated by the markings on the table. Your space diagram should now represent the arrangement of the force table.

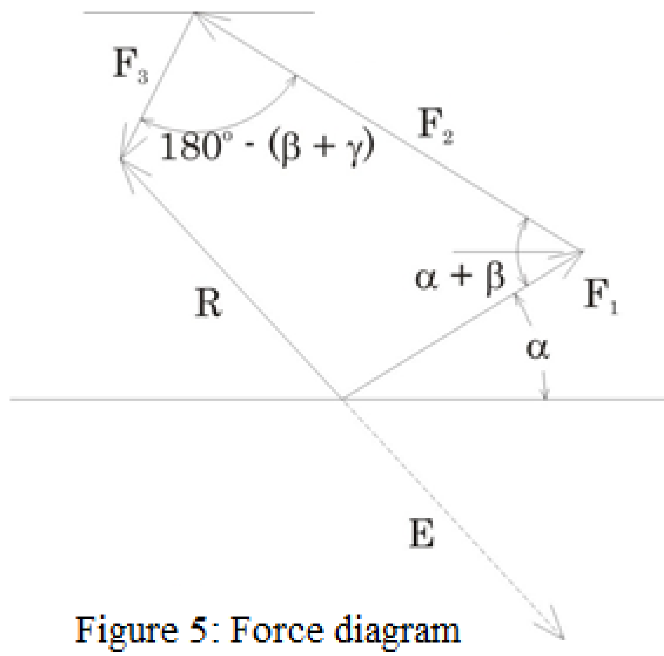

Figure 5: Force diagram

If all your calculations and adjustments have been properly made, the ring should be suspended by the strings and touch the pin at no point. If these conditions are not fulfilled, find your error and correct it. If your system is balanced, remove the pin, displace the ring a few centimeters from the center and release it. Note what happens. See Question 1.

In performing this laboratory in both part I and II, you should first determine the magnitude of the resultant, E and its direction prior to doing any experiments. You should compare your experimental values for the magnitude of E and the angle (relative to 0 degrees) to the theoretically predicted ones. The values you determine should be in close agreement, otherwise you have performed the laboratory incorrectly. Check your data and the computations before leaving the laboratory.

Part II: Rectangular resolution and equilibrium of coplanar forces

On another piece of plain white paper, redraw Figure 4, but this time to scale, letting 1.0 cm correspond to the force due to 10 grams. See Figure 6.

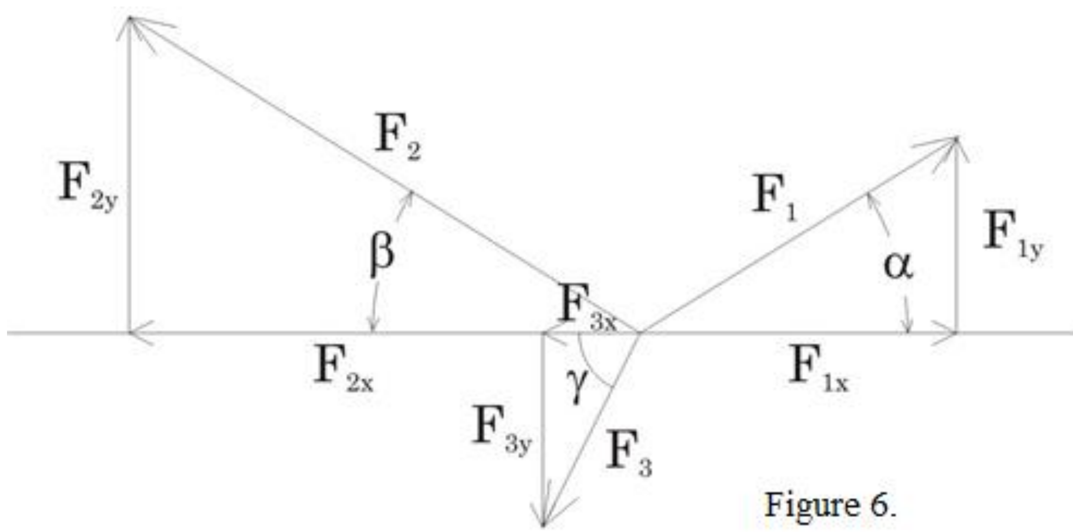

Figure 6.

1. Draw a line from the end of $\mathbf{F_1}$ that is perpendicular to the x axis as shown in Figure 6. The length of this line represents F_{1y}, the y component of $\mathbf{F_1}$; and the point where it crosses the x axis marks F_{1x}, the x component of $\mathbf{F_1}$. Measure with the centimeter ruler the magnitudes of these two components.
2. Repeat this procedure with forces $\mathbf{F_2}$ and $\mathbf{F_3}$. Take care to note which forces have negative x-components and which have negative y-components.
3. Add up the x-components of $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$. We will call this ΣF_x.
4. Add up the y-components of $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$. We will call this ΣF_y.
5. Since these six components may now take the place of the original three forces, it is possible to balance the original three forces by balancing their components. Check this experimentally by means of the force table as follows: clamp three pulleys on the force table, one to represent the equilibrant as determined in Part I, one on the 0 - 180° axis to represent the sum of the x components, ΣF_x, and one on the 90 - 270° axis to represent the sum of the y components ΣF_y. Hang the appropriate weights on the weight hangers and test for equilibrium. The arrangement of the table should be similar to Figure 7.

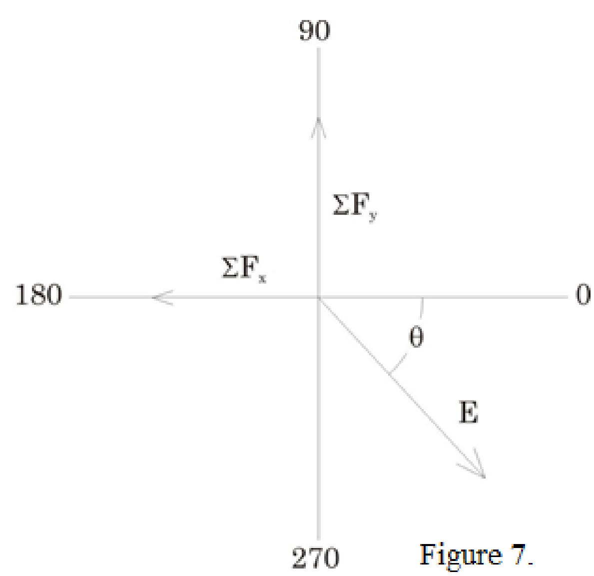

Figure 7.

Computations

1. Compute by trigonometry, using the component method, the magnitude of the resultant of the three forces. This is also the magnitude of the equilibrant.
2. Compute by trigonometry, using the component method, the angle θ for the equilibrant.
3. Compare the computed magnitude and angle with those obtained graphically, and compute the percent discrepancy (“error”) for each.
4. Compare your experimental values for the magnitude of E and the angle (relative to 0 degrees) to the theoretically predicted ones. The values you determine should be in close agreement, otherwise you have performed the laboratory incorrectly. **Check your data and the computations before leaving the laboratory.**

Questions

1. In Part I, when the ring was in equilibrium, the forces and their directions have certain values. If the ring is now displaced, do the forces have the same magnitudes as before? Are the directions the same as before? Referring to your two preceding answers; explain whether or not the sum of the forces is still zero for the displaced ring.
2. State the first condition for equilibrium in two ways: (a) as illustrated by Part I and (b) as illustrated by Part II.

Data sheets

Your drawings are data sheets for this experiment.