

INTRODUCTION TO LABORATORY EXPERIMENT AND MEASUREMENT

Purpose

- To take some simple measurements to use for statistical analysis
- To learn how to use a Vernier caliper and a micrometer screw gauge.
- To learn about how straight line plots may be used to extract physical quantities by comparison of experimental data with a theoretical expression

Theory

This lab is comprised of two parts:

Part I. Measurements of length and the use of statistics to analyze a set of data

The theory relating to this part of the lab is that of statistical analysis and *uncertainty*. You may have had to perform statistical analyses in other classes at the College. There is a shared purpose visiting the topic of statistical analysis in several different contexts: measured experimental data can depend on many factors and may not be exactly repeatable - hence the idea of uncertainty. In order to estimate not only the value of a quantity but the uncertainty in its measurement, we perform several measurements and then calculate the spread of the data. For more on this topic, see the section entitled "Interpretation of Data" in the *Introduction to The Physics Laboratory*. A brief summary is as follows:

Any result of a measurement of a quantity "X" can be presented as a range around the best estimated value and is written as

$$X = x \pm \Delta x .$$

Here, x is normally the average of several measured values while Δx is the *standard deviation*, the estimate of uncertainty in the quantity X. For example, when you measure the inner diameter, D_{inner} of a hollow block during this lab, you will find that the values of this quantity vary slightly along the length of the block. So what value should be stated as the inner diameter? The answer is that we quote the average value *and* the estimate of uncertainty, which is the *standard deviation*. Uncertainty comes from several sources, but there are two general forms of uncertainty, or error.

Firstly, there are "systematic" errors, that bias a measurement in one direction. For example, if a mass balance is not reset to zero, all masses measured on it thereafter will be incorrect by the same amount. Secondly, there are "random" errors, which can arise from measurement difficulties, fluctuations in temperature, pressure and so on. We try to minimize both types of error as much as possible. In your lab reports this semester, you should not only state what *kind* of error is present in your experiment, but also the *source* of the error. In this way, your lab reports will be as particular to your experiment as possible.

From multiple measurements of the same quantity, we may find the *average* or *mean* value. for example, if the above quantity "X" is measured N times, the average value of the quantity is

$$x = \frac{\sum_i x_i}{N} .$$

Sometimes the average quantity is denoted by $\langle \rangle$ brackets (e.g. $\langle x \rangle$) or by a line over the symbol: \bar{x} .

To estimate the uncertainty or standard deviation in the same set of N measurements, we perform the following calculation:

$$\Delta x = \sigma = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{N-1}}$$

The above calculation means that each measurement, x_i is compared to the average value, \bar{x} , and the difference is squared. *Note that this means that positive and negative differences from the mean both count in the sum, and so the greater the spread of results of a measurement, the larger the standard deviation.* The sum of the squares of these differences is then divided by the number of measurements minus one and the square root of the whole result is taken. In the above formulae you will see the symbol for standard deviation, σ . This is sometimes used; alternatively we may state the standard deviation in x as Δx . As stated above, the result of this analysis is that we may quote our estimate of and the uncertainty in it by stating that $X = \bar{x} \pm \Delta x$.

In Part I of this lab, you will make measurements of diameters that will lead to a statement of average diameter and standard deviation. *Note: the choice of letter (X , x etc.) here is just for demonstration purposes.*

Part II. Use of straight line plots to extract physical quantities

Many of the general physics I lab experiments will ask you to graph your data in a particular way and from precalculus, you know that the equation:

$$y = mx + c$$

can be plotted as "y versus x", meaning that "y" goes on the vertical axis while "x" goes on the horizontal axis. (There are also known as the "ordinate" and "abscissa", respectively.)

Not all equations just involve "x" however! Let's take the example of Lab 2: a ball falling under gravity. The distance that the ball falls is given the symbol d while the time that the ball takes to fall is t . The theoretical equation that links d and t , is:

$$d = \frac{1}{2}gt^2,$$

where $g = 9.8 \text{ m/s}^2$ is the gravitational acceleration of objects on the Earth's surface. Here is an example of Lab data:

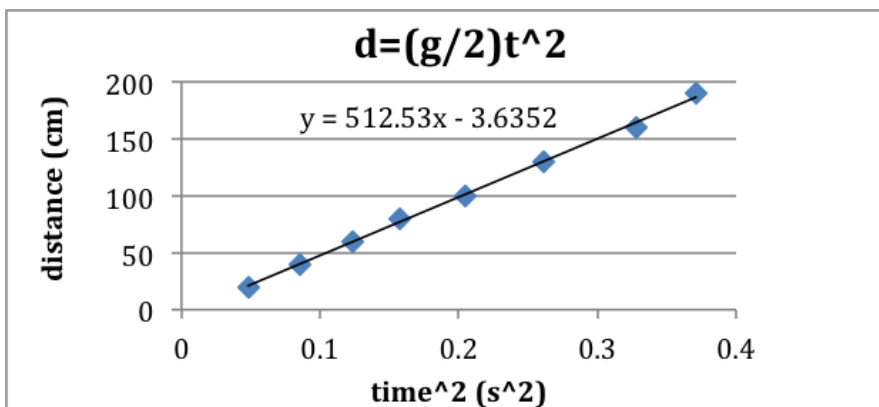


Figure 1: An example of data for a free fall experiment, plotted so as to obtain a straight line graph.

In Lab 2, you are asked to find g by using your data for different values of d and t . How do we do this? Looking at the above equation and comparing with " $y = mx + c$ " we see that **if we plot d vs t^2 then we will get a straight line, with gradient $\frac{g}{2}$** . The best fit line through the data points should also go through the origin since the constant term, " c " is zero.

In the above graph, the distance was given in centimeters (100 cm = 1 meter). Therefore, the gradient is an experimental estimate of $g/2 = 512 \text{ cm/s}^2$. [Note the units results from those used in the plot.]

Extracting the desired quantity from the graph slope.

In the above example, we have estimated from the graph slope that: $g/2 = 512 \text{ cm/s}^2$ by examining the trendline produced (in this case) by Microsoft Excel. (For more on the use of Excel, please see the *Introduction to The Physics Laboratory* document.) You may also extract the slope by simply calculating the gradient of the best fit straight line.

So, if we now want to find the value of g , we just use this result to say that

$$g = 2 \times 512 \text{ cm/s}^2 = 1025 \text{ m/s}^2.$$

The actual number of significant figures used to express the result will depend on the accuracy of the measurements that led to the plot. For more on significant figures, see chapter 1 of the course textbook.

In Part II of this lab, you will examine more examples of equation rearrangement to find out how the theoretical equation that describes an experiment can be used to construct a straight line graph from which we extract a slope value that tells us some particular physics.

Apparatus

One hollow block; one steel ball; a Vernier caliper; a Micrometer; a triple beam balance

Description of Apparatus

In the experimental section of this lab, you will use a Vernier caliper to estimate the volume of material in a hollow block of aluminium. From a separate measurement of the block's mass, you'll then calculate the density of aluminium. You will use a micrometer to make a measurement of the diameter, and hence volume, of a steel ball. As in the case of the hollow block, you'll use a measurement of the mass of the ball to calculate the density of steel.

Vernier caliper

The Vernier caliper is an instrument that is often used to measure external and internal dimensions of

objects with precision up to one hundredth of a centimeter (or equivalently, one tenth of a millimeter). It uses a *Vernier scale*, pioneered by French mathematician Pierre Vernier in the 17th century. The purpose of the Vernier scale is to allow a good reading of a measurement that lies between two tick marks of a main scale. The way that this is done is by having a second, Vernier, scale that is used along with the main scale to provide additional accuracy. The graduations on the secondary scale have a different spacing to those on the main scale, such that N graduations on the secondary scale cover N-1 graduations on the main scale.

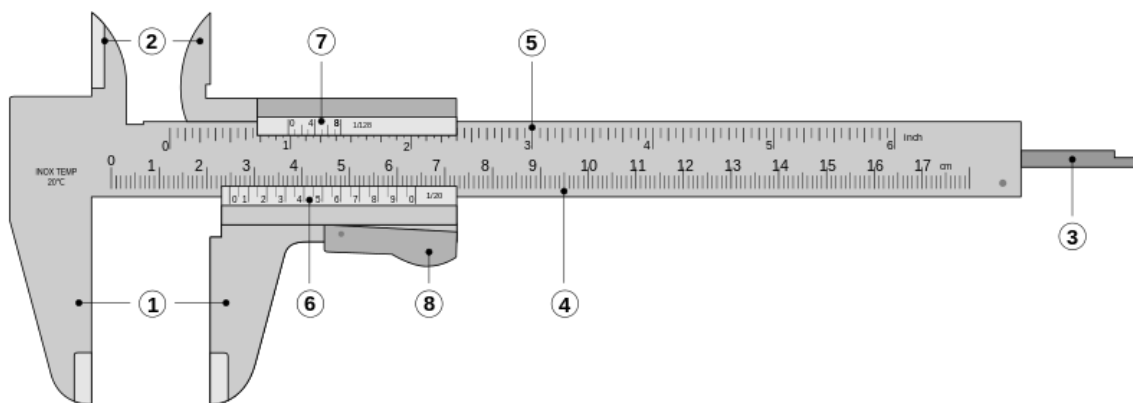


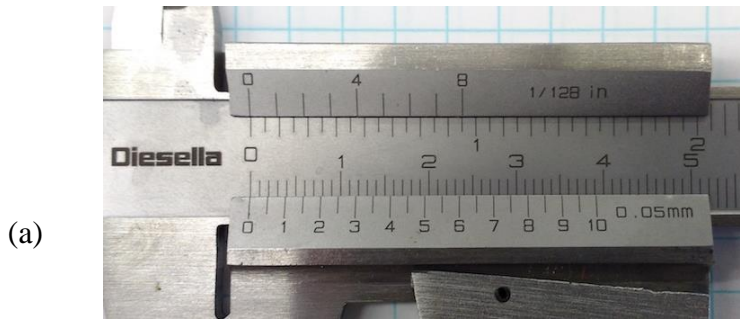
Figure 2: a typical Vernier caliper. The numbered parts are: (1) The large, external jaws: used to measure external diameter or width of an object; (2) The small internal jaws: used to measure internal diameter of an object; (3) Depth probe: used to measure depths of an object or a hole (4) Main scale: this scale is marked every mm; (5) a Main scale: this scale is marked in inches and fractions; (6) a Vernier scale, which gives interpolated measurements to 0.1 mm or better; (7) a Vernier scale, which gives interpolated measurements in fractions of an inch; (8) Retainer: used to block movable part to allow the easy transferring of a measurement. Image and caption courtesy of Wikimedia Commons:

https://commons.wikimedia.org/wiki/File:Vernier_caliper.svg

There is a large demonstration version of a Vernier scale in the laboratory. However, here is a brief guide to reading a measurement from a caliper. Remember: as with many measurements, you can also check your reading with, for example, a ruler, to make sure you have not a very large error if you are using this device for the first time.

First, look for where the zero on the secondary, Vernier scale lies against the main scale. In Figure 3(a), the zero on the secondary (Vernier) scale sits at zero on the main scale. This tells us that the reading is 0.0 cm. In Figure 3(b), the zero on the Vernier scale sits between 0.3 and 0.4 cm but we don't know just from whether the next decimal place is 0.31, 0.35, 0.37 cm, etc. For that second decimal place we look for *where the secondary scale lines up with a division on the main scale*. Again Figure 3(b), this occurs at the the 8.5 mark on the Vernier scale. So the reading is 3.85 cm. Try reading the Vernier scale on Figure 3(c) for yourself and see if you agree that the reading is 1.735 cm. The error in each reading is ± 0.005 cm.

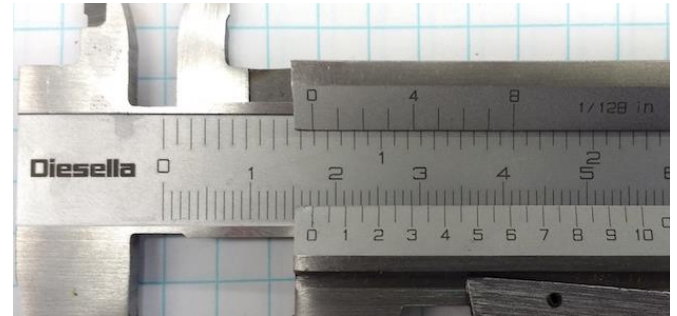
For further examples of how to read the Vernier scale, and some practice where you can test yourself, see: <http://www.technologystudent.com/equip1/vernier3.htm>



(a)



(b)



(c)

Figure 3: Examples of Vernier caliper readings. In each picture, the main scale is on the body of the caliper while the lower scale is the Vernier scale, measuring to 0.005 cm. The readings are: (a) 0.00 cm; (b) 0.385 cm; (c) 1.735 cm (all ± 0.005 cm)

Micrometer

The micrometer works on a different basis, but can achieve even higher precision by interpreting between secondary scale marks. It is also supplied with a Vernier secondary scale (also known as a nonier scale) and so you can use a similar approach to make readings of the diameter of a steel ball at different locations. Examples of readings are given below, where interpretation between the Vernier scale divisions allows us to state a reading to the nearest 0.0001 cm, i.e. ± 0.0001 cm.



(a)



(b)

Figure 4: Micrometer gauge readings. The Vernier scale on the screw can be interpreted to give readings of (a) 0.3651 ± 0.0001 cm; (b) 1.0411 ± 0.0001 cm.

Procedure

Part I

1. Take the **hollow block** and use the Vernier caliper to measure its *inner* diameter in millimeters. Use Table 1a of the data sheet to record the different measurements that you take. You can rotate the hollow block relative to the calipers to perform different trials, at either end of the block.
2. Measure the *outer* diameter of the hollow block at 5 places along its length. Write down the location of the measurement and the value of the diameter that you measure in Table 1b.
3. Measure the *outer length* of the hollow block 5 times using the Vernier caliper. Record these values in Table 1c.
4. Use the depth probe (the part of the Vernier caliper that is labelled "3" in Figure 3) to measure the *inner length* of the hollow block at 5 locations. Record your values in Table 1d.
5. Use the mass balance to measure the mass of the hollow block.
6. Take the **steel ball** and use the micrometer to measure its diameter in millimeters, at 5 locations. Record your values in Table 2. Measure its mass using the triple beam balance.

Part II

1. See the *Computation and analysis section* below.

Computation and analysis

Part I

1. For all measurements in (1) and in (2) of Part I above, calculate the average value of the diameter and the standard deviation of the (inner or outer) diameter that you have measured.
2. For each case, state the value of the diameter that you have found in the form
$$D = d \pm \Delta d$$
where D is the diameter that you are measuring, d is its average value and Δd is its standard deviation.
3. From the measurements of the inner and outer diameter and inner and outer length that you made using the Vernier caliper, estimate the *volume* of material that makes up the block. HINT: subtract the volume of the inner cylinder of air from the outer cylinder of solid material.
4. From the mass measurement and your estimate of the volume of the block, calculate the density (mass/volume) of the material that the block is made of. State the units of your answer. Compare your answer to the density of aluminium: 2.70 g/cm^3 .
5. From your measurements of the steel ball (Table 2), state the diameter of the ball you have found in the same form as given above ($D = d \pm \Delta d$). Your value of Δd in this case should be lower. Why?
6. From your measurements of diameter and mass for the steel ball, work out the density of

steel. Compare to the density of steel: 7.60-7.80 g/cm³. (Steel is an alloy that can vary in composition depending on its function and so a range of densities is possible.)

Part II

You can see in many of the general physics labs that we **rearrange the theoretical equation that describes the experiment** to find a **straight line graph from which we extract a slope. From the slope that we find, some desired quantity** (the acceleration of gravity, the spring constant of a spring, the length of a pendulum etc.) can be calculated.

So, **if you are asked to plot "B versus A", this means that "B" goes on the vertical axis, and "A" goes on the horizontal axis.** In many of the labs, you will be asked to calculate the slope (gradient) of a best fit straight line through your experimental data.

A worked example

If we have a theory that states that $F = \frac{Mv^2}{R}$

What would be the slope of the graph of v^2 vs R ? For this, we have to rearrange the equation:

$$v^2 = \frac{FR}{M}$$

So, if we plotted v^2 on the vertical axis and R on the horizontal axis, we expect a straight line graph with slope (or "gradient") F/M . From that slope value we can find F if we already have M , or vice versa.

Questions

(Answering these questions provides further examples of extracting the theoretical slopes of straight line graphs)

a) If we have a theory that states that $F = \frac{4\pi^2 MR}{T^2}$

What would be the slope of a graph of T^2 vs M ? And what units would the slope have, if T is a time measured in seconds and M is a mass measured in kg?

b) If we have a theory that states that $T = 2\pi\sqrt{\frac{m}{k}}$

What would be the slope of a graph of T^2 vs m in this case? And what units would the slope have, if T is a time measured in seconds and m is a mass measured in kg?

c) If we have a theory that states that $T = 2\pi\sqrt{\frac{l}{g}}$ (where l is a length in meters).

What would be the slope of a graph of T^2 vs $\frac{1}{g}$ in this case? And what units would the slope have, if T is a time measured in seconds and l is measured in meters, and g is measured in units of m/s².

More Questions

1. When measuring the inner and outer diameters of the hollow block, what were the main sources of experimental error? Group these errors into those that are systematic and those that are random.
2. How could you improve the measurement of the block diameters? In particular, consider how to improve the accuracy of the average value of diameter and how to reduce the uncertainty in the measurement.
3. To how many significant figures can you state the volume of material that makes up the block when calculated with measurements from the Vernier caliper? To how many significant figures can you state the volume of the steel ball when calculated with measurements from the micrometer screw gauge?
4. State the density of the hollow block in two different units: g/cm^3 and kg/m^3 .

Data Sheet for Lab 1

Name of student:

Date:

Hollow block, measured using the Vernier caliper.

Table 1a

Trial number	Inner diameter (cm)	Square of inner diameter (cm ²)

Table 1b

Trial number	Outer diameter (cm)	Square of outer diameter (cm ²)

Table 1c

Trial number	Outer Length (cm)	Square of length (cm ²)

Average inner diameter: _____ cm

Average outer diameter: _____ cm

Average outer length of block: _____ cm

Standard deviation:

Standard deviation:

Standard deviation:

Table 1d (hollow block)

Trial number	Inner length (cm)	Square of inner length (cm ²)

Table 2: Steel ball, measured using the micrometer

Trial number	Diameter (cm)	Square of diameter (cm ²)

Volume of material in block: _____ cm³.

Mass of hollow block: _____ g.

Density of hollow block: _____ g/cm³.

Average diameter of the ball: _____ cm

Standard deviation:

Volume of material in steel ball: _____ cm³.

Mass of steel ball: _____ g.

Density of steel ball: _____ g/cm³