Newton's Laws of Motion

Purpose

To apply Newton's laws of motion for a cart moving on a track with constant acceleration. We will consider three experiments,

I. a cart self-propelled by a fan,
II. a cart on a horizontal track and pulled by a rope attached to a hanging mass, and
III. a cart *on an inclined track* pulled by a rope attached to a hanging mass.

Theory

In this lab, we will use Newton's laws of motion to find the acceleration of a cart for three different cases mentioned above. A motion sensor will be used to collect the data for the position and velocity with respect to time while the cart is moving. From the graphs of position and velocity versus time, we will determine the acceleration of the motion experimentally. In each case, we will compare the acceleration of the cart measured in the experiment with the corresponding acceleration calculated using Newton's laws of motion.

Part I. A cart self-propelled by a fan

Consider a cart on a frictionless track as shown in the Figure 1. A fan is attached to the cart. When the fan is on, air is blown out to the left in the figure. We are assuming that the positive x-axis points to the right. The fan applies a force $\vec{F}_{FAN-on-AIR}$ to the air along the direction of negative x-axis.

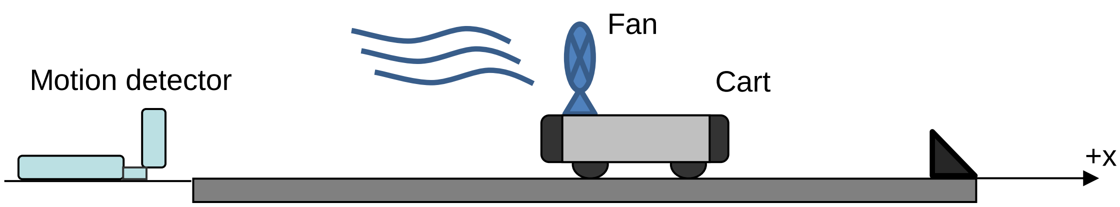


Figure 1: Setup for part I.

Newton's third law implies that the air must apply a force $\vec{F}_{AIR-on-FAN}$ on the fan and that $\vec{F}_{AIR-on-FAN} = -\vec{F}_{FAN-on-AIR}$. As a result, the cart (with the attached fan) will experience a net force along the +x direction, $\vec{F}_{FAN-on-AIR} = F\,\hat{x}$, where F is the magnitude of force and $\hat{x}$ is a unit vector along the +x direction.

From Newton's second law, $\vec{F}_{net} = m\vec{a}$, it follows that the effect of turning the fan on is to produce a (constant) acceleration of the cart along the x-direction, given by

$$a_{cart,x} = a = F/m, \qquad (1)$$

where m is the mass of the cart including the fan.

Part II. A cart on a horizontal track and pulled by a rope attached to a hanging mass

Consider a cart of mass *m* moving on a horizontal 'frictionless' track. The cart is pulled by a rope which is passing over a frictionless pulley and attached to a hanging mass.

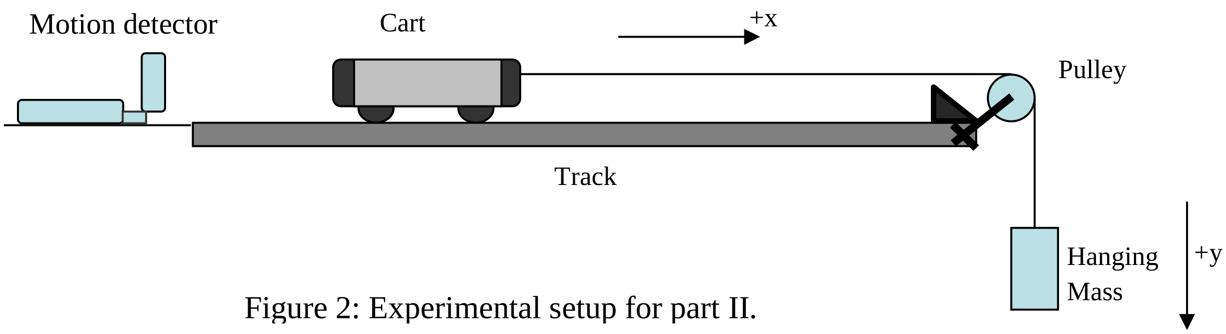


Figure 2: Experimental setup for part II.

Free body diagrams are shown in Figure 3 to determine the acceleration of the cart. Figure 3a is the free body diagram for the cart. We assumed that the positive x-axis is parallel to the track and points to the right. Since the cart is on a horizontal track, the normal force, ***N*** balances the weight of the cart, $\boldsymbol{W_{cart}}$. The only force acting on the cart *along the direction parallel to the track* is the tension of the rope, *T*. Hence, from Newton's second law, we obtain for the x-axis:

$$T = m\, a_{cart,x} \tag{2}$$

where $a_{cart,x}$ is the acceleration of the cart along the x direction and T is the tension of the rope.

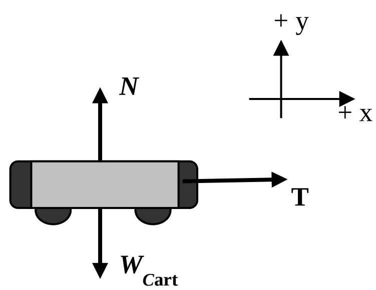


Figure 3a. Free body diagram of cart.

The tension can be obtained by applying Newton's second law to the hanging mass. Consider the free body diagram for the hanging mass shown in Figure 3b,

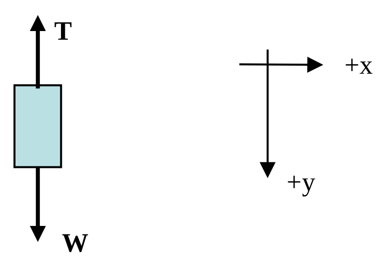


Figure 3b. Free body diagram of the hanging mass.

where we define the (+y)-axis downwards. Then, it follows that

$$Mg - T = M\, a_{mass,y} \tag{3}$$

where $W = Mg$ is the weight of hanging mass and $a_{mass,y}$ is the acceleration of the hanging mass along the y-axis. Since the cart and the hanging mass are connected by a rope that passes over a frictionless pulley, the acceleration of the cart and the hanging mass are equal in magnitude:

$$a_{cart,x} = a_{mass,y} = a \tag{4}$$

Therefore, from Eqns. (2), (3) and (4) we obtain for the (constant) acceleration of the cart:

$$a = \frac{M\,g}{m+M} \tag{5}$$

and the tension is given by Eqn. (2),

$$T = \frac{M\,m\,g}{m+M} \tag{6}$$

Part III. A cart on an inclined track and pulled by a rope attached to a hanging mass

In this part of the experiment a cart moves on an inclined track as shown in Figure 4. The setup is similar to part II with the track forming an angle θ with the table. As in part II we assume that there is no friction between the track and the cart.

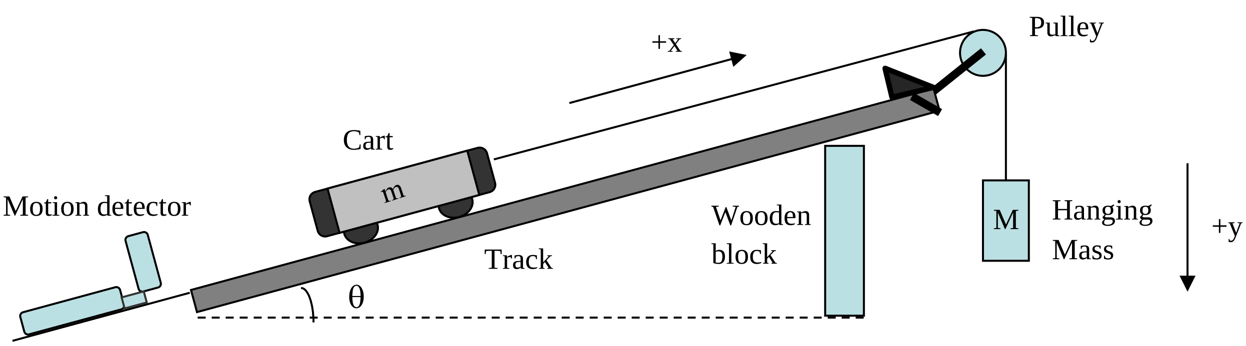


Figure 4: Experimental setup for part III.

When applying Newton's second law to the cart, it is convenient to define the (+x)-direction parallel to the track, as shown in Figure 4. With this choice, there are two forces acting on the cart that have non-zero components on the x-axis. One of these forces is the tension, which is parallel to the x-axis, i.e., $T_x = T$. The second force is the weight, which points downward. The x-component of the cart's weight force is $W_{cart,x} = -m\, g$ sinθ. Therefore, Eq. (2) is now replaced by

$$T - m\, g\, \sin\theta = m\, a_{cart,x} \quad \textbf{(7)}$$

If we define the (+y)-direction for the hanging mass to be downwards, as shown in Fig. 4, then Eqs. (3) and (4) hold for the hanging mass of Figure 4 as well. Therefore, combining Eqs. (3), (4), and (7) we obtain for the (constant) acceleration of the cart:

$$a_{cart,x} = a = \frac{g\,(M - m\,sin\theta)}{m+M} \quad \textbf{(8)}$$

and the tension is given by Eqn. (7),

$$T = M\, m\, g\, \frac{(1 + sin\theta)}{m+M} \quad \textbf{(9)}$$

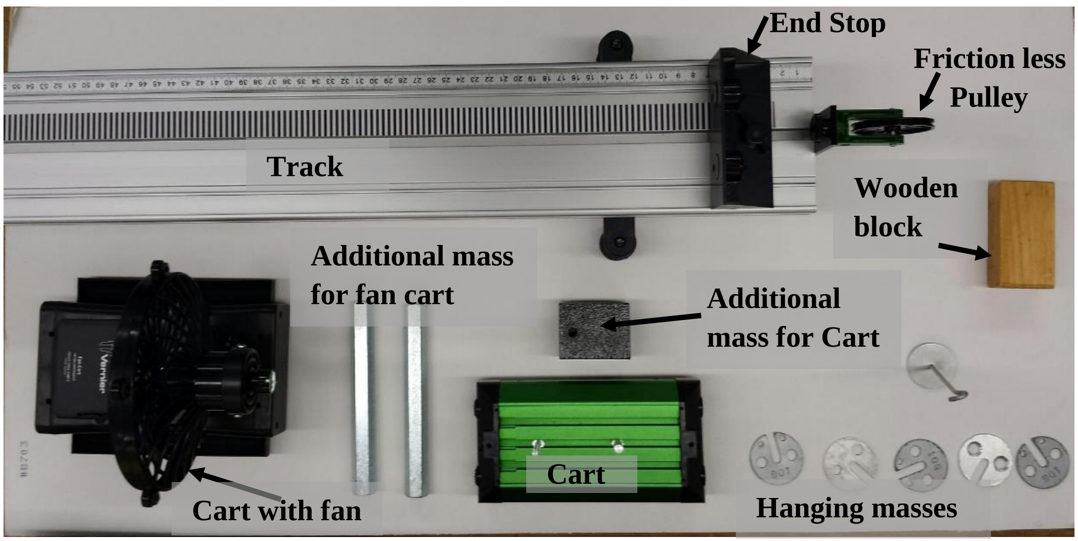
Apparatus

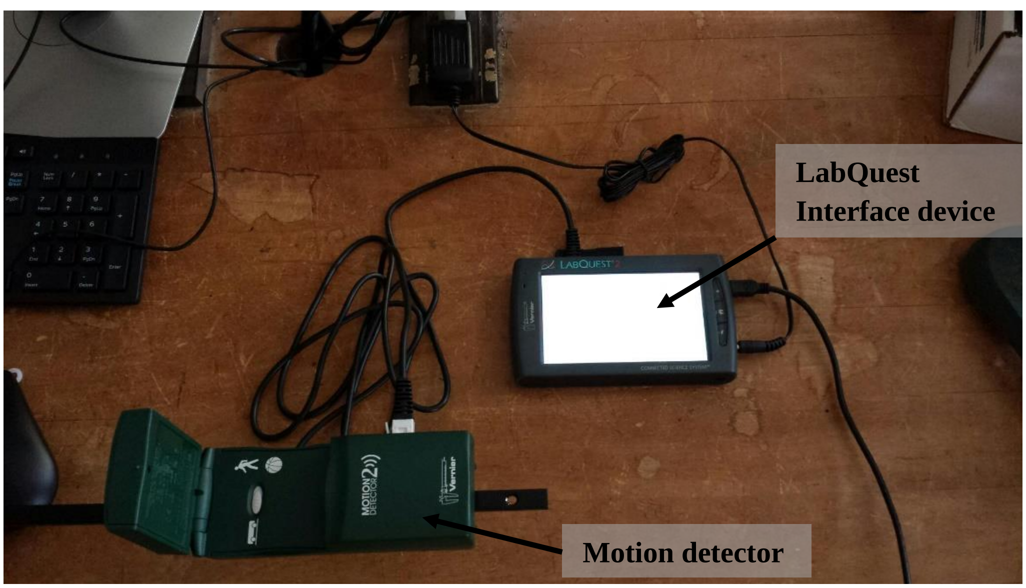
Cart (green) and mass attachable to the cart, cart with fan (black) and attachable metal bars, track, hanging masses, pulley, rope, wooden block, scale, ruler, ultrasonic motion detector, Vernier LabQuest interface, and a computer with Logger Pro software.

Description of Apparatus

Vernier cart and track are used in this experiment. The cart and track are designed to have negligible friction. A motion detector connected to a computer is used to collect the data while the cart is moving (see picture below). The motion detector emits ultrasonic (higher than audible frequency sound wave) pulses. These sound pulses bounce on the cart and return to the motion detector. The motion detector measures the time taken by the pulses to return and, based on the known speed of sound, it determines the position of the cart from the motion detector. The motion detector makes 30 distance measurement every second.

The motion detector is connected to a Vernier LabQuest interface device which is then connected to a computer. We will use Logger Pro software in the computer for the measurement and analysis. The program collects the data of position versus time. Based on these data, it also calculates the velocity. These data are displayed in a table. It also plots the graphs of position versus time and velocity versus time. We will analyze these graphs to determine the acceleration of the motion.

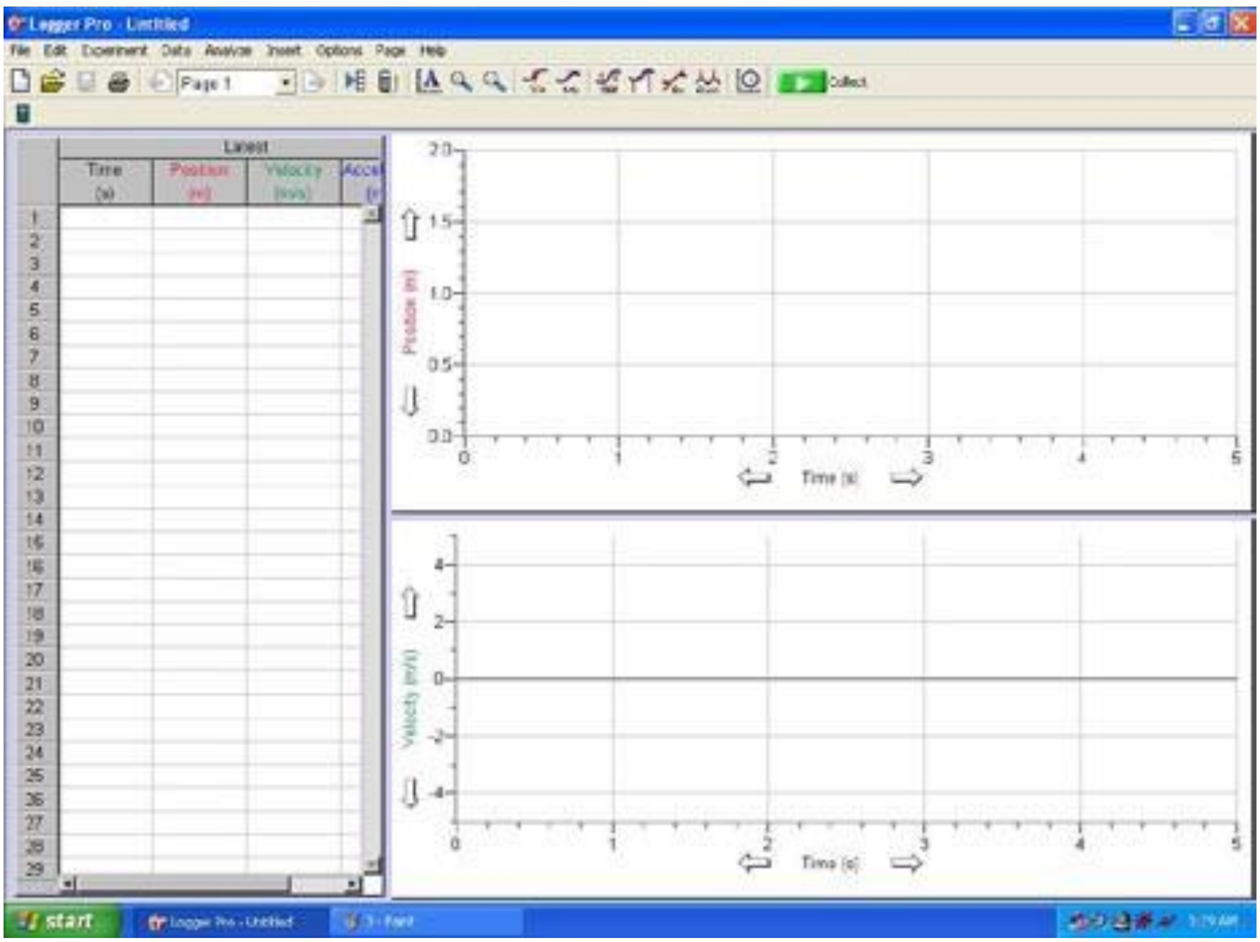




Procedure

Part I. A cart self-propelled by a fan

1. Measure the mass of the cart (*m*) with the fan using the digital scale and record in Table 1.
2. Check if the track is leveled. You can easily check if the track is leveled or not by placing a cart on the track. If the cart stays at rest even if you move the cart to another spot, it is leveled. If the track is not leveled, the cart starts moving. You should then level it by adjusting the screws under the track.
3. Connect the LabQuest interface device to the computer and to the motion detector (see picture above). Turn on the LabQuest interface device.
4. Computer should be already turned on. Open **"Logger Pro"** program from desktop. New window opens with two blank graphs (position versus time and velocity versus time) and a blank table (time, position, velocity) as shown below.



5. Practice how to use the computer software. Using the left button on the mouse, click on the "**Collect**" button to start taking data. There is a slight delay (1-2 seconds) before data collection actually starts. The position velocity values will be shown in the table and plotted in the graphs. Collection will stop automatically after 5 seconds have elapsed.
6. Hold the cart on the track ~20 cm from the motion detector. Turn on the fan and set the power level to "1").

7. Now, click on the **"Collect"** button to start taking data and release the cart to set it in motion. The cart should accelerate along the track while the computer plots graphs of distance versus time and velocity versus time. **Turn off the fan**. If the graphs do not look good (consult with your instructor) then you must repeat the measurement. To collect new data, simply click on collect and the program will over write and erase the previous data. When you have good data, **save it in a file** with a name of your choice.

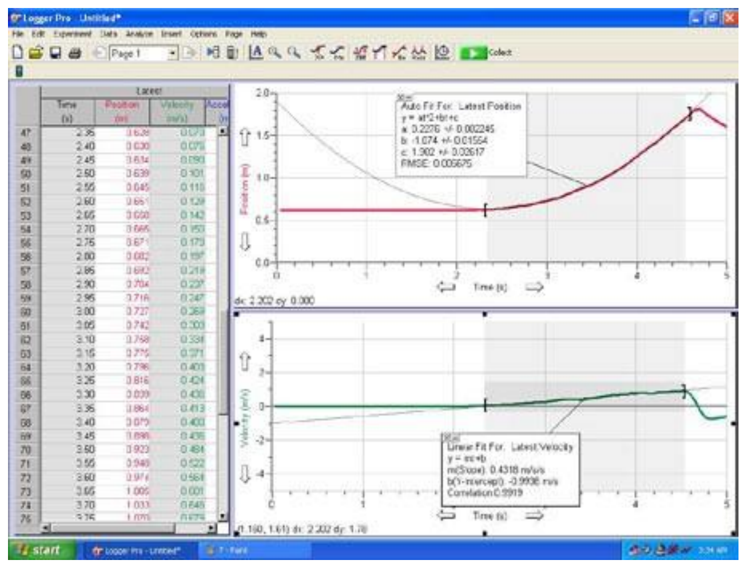
   Note: *Make sure that no one is standing in front of the motion detector. If the sound waves are reflected from objects other than the cart then you will get wrong data.*

8. When you have graphs that look as good as possible, ask your instructor to confirm that you are ready to analyze your data. There may be spurious motions at the beginning or end of your experiment that you can eliminate (see the screen snapshot below). In the velocity versus time plot, highlight the region in the graph with the data to be analyzed. In the menu bar, click **"Analyze"** and select **"Linear Fit"**, which gives you the best straight line fit to the data (see plot below). A small window will also appear with the values for the slope (m) and y-axis intercept (b). The slope is the acceleration of the cart. Record it in Table 1.

9. In the position-time plot, highlight the region in the graph with the data to be analyzed. Go to the menu bar and click **"Analyze", select "curve fit" and choose "Quadratic"**. This will give you the best fit curve to the data by a quadratic equation $y=Ax^2+Bx+C$. The values of the coefficients A, B, and C will be displayed in a box. (see plot figure). The acceleration of the cart is A/2, record it in Table 1

10. Print the graph for your report. You may also save the data and plot the graphs using Excel.

11. Now you are going to add load on the cart. Measure the mass of the two metal rods and load them on the both sides of the cart and repeat steps 6-9 (see Table 1).



Part II. A cart on horizontal track and pulled by a rope attached to a hanging mass

In this part of the experiment, we will use a rope connected to a hanging mass to pull the cart.

1. Replace the black cart with the fan by the green cart. Use the digital scale to measure its mass, *m* and record this value in Table 2.
2. Attached a rope to the cart. Run the rope over the frictionless pulley and attach the other end of the rope to a hanging mass, M = 0.01 kg (as shown in Fig. 2). Hold the cart in a fixed position ~20 cm from the motion detector.
3. Repeat steps 7 – 9 of part I.
4. Repeat steps 14 and 15 using a hanging mass M = 0.02 kg.
5. Load the cart with the attachable mass and repeat steps 14-16. When finished, you will have a total of 4 measurements (see Table 2).

Part III. A cart on an inclined track pulled by a rope attached to a hanging mass:

1. Set up the equipment as shown in Figure 4 using the shortest side of the wooden block to set the angle, θ. Determine the angle of inclination, θ by measuring the heights and length of the track. The hanging mass will be fixed to M ~ 0.20 kg. **[Note: Based on the angle of inclination, the hanging mass might need to be adjusted].**
2. Hold the cart in a fixed position ~20 cm from the motion detector and repeat steps 7 - 9 of part I.
3. Repeat these steps using the medium and longest sides of the wooden block to set the higher angles, θ. When finished, you will have a total of three measurements (see Table 3).

Computation

(I) A cart self-propelled by a fan

Using the data collected in Table 1, check if your observation validates, $a \propto \frac{1}{m}$.

Calculate the force exerted by the fan on the cart.

(II) A cart on horizontal track and pulled by a rope attached to a hanging mass

Complete the Table 2. Calculate the theoretical value for the acceleration using equation (5). Other quantity of this table is the "% error of a, which tells you how close your experimental results compared to the prediction from Newton's laws.

% error in $a_{cart,x} = 100 \, \frac{|(eperimetnal\ a_{cart,x}) - (theoretical\ a_{cart,x})|}{(theoretical\ a_{cart,x})}$

(III) A cart on an inclined track pulled by a rope attached to a hanging mass

Complete Table 3.
Plot a graph of a versus $sin\theta$. Is this a linear graph?
Find the slope of the best straight fitting line.
Compare this value with the corresponding theoretical slope = $\frac{m\,g}{m+M}$ (see Eq. (8)).

Questions

1. (a) Draw a free-body diagram for the cart in Fig. 4. Obtain Eqn. (7).
   (b) Calculate the normal force of the cart.
   (c) Obtain Eqns. (8) and (9).
2. The cart in the setups of Figs. 1, 2, and 4 should have constant acceleration, a. Make 3 schematic diagrams of (i) acceleration, (ii) velocity, and (iii) position as function of time for the cases of an object moving with constant (a) $a > 0$ and (b) $a < 0$.
3. In part I of the experiment, should the force on the cart (F) depend on the mass of the cart ? Does your answer agree with the results with Table I?
4. In part II and II of the experiments, what would be the acceleration of *hanging mass* if the rope suddenly breaks?
5. In part III of the experiment, what would be the *cart*'s acceleration if the rope suddenly breaks? Interpret your result.

Data Sheet

Date experiment performed:

Name of the group members:

I. A cart self-propelled by a fan: **Table 1**

m [kg]	a_{cart} [m/s^2]			$F = m\,a_{cart}$ [N]
	From *v* vs *t* graph	From *x* vs *t* graph	Average	

II. A cart on horizontal track and pulled by a rope attached to a hanging mass: **Table 2**

m [kg]	*M* [kg]	a_{cart} [m/s^2]					*T* [N] [Eqn. (6)]
		Experiment			Theory [Eq.(5)]	% error	
		From *v* vs *t* graph	From *x* vs *t* graph	Average			
	0.01						
	0.02						
	0.01						
	0.02						

III. A cart on an inclined track, pulled by a rope attached to a hanging mass: **Table 3**

m [kg]	*M* [kg]	θ [deg]	sin(θ)	a_{cart} [m/s^2]					*T* [N] [Eqn. (9)]
				Experiment			Theory [Eq.(8)]	% error	
				v vs *t* graph	*x* vs *t* graph	Average			

Slope of the best straight fitting line of the graph of *a* versus $sin\theta$: