SIMPLE PENDULUM AND PROPERTIES OF SIMPLE HARMONIC MOTION

Purpose

a. To investigate the dependence of time period of a simple pendulum on the length of the pendulum and the acceleration of gravity.

b. To study properties of simple harmonic motion.

Theory

A simple pendulum is a small object that is suspended at the end of a string. "Simple" means that almost all of the system's mass can be assumed to be concentrated at a point in the object. We will use a metal bob of mass, *m*, hanging on an inextensible and light string of length, *L*, as a simple pendulum as shown in Figure 1. When the metal bob is pulled slightly away from equilibrium and released, it starts oscillating in a simple harmonic motion (SHM). The restoring force in this system is given by the component of the weight ***mg*** along the path of the bob's motion, $\boldsymbol{F = -mg \sin\alpha}$ and directed toward the equilibrium. For **small angle**, we can write the equation of motion of the bob as

$$a = -g \sin\alpha = -g \frac{x}{L} \qquad (1)$$

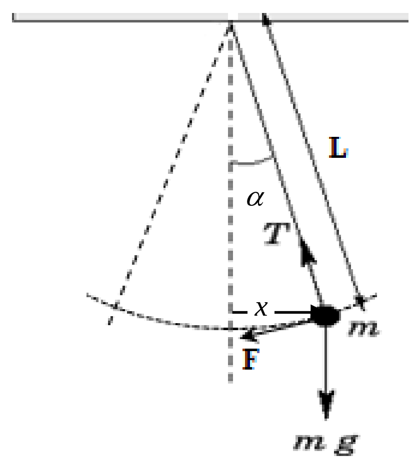


Figure 1. A simple pendulum

In a simple harmonic motion, acceleration is directed towards the equilibrium and proportional to the displacement. The acceleration (a) and displacement (x) are given by

$$a = -\omega^2 x \qquad (2)$$

The displacement (x) varies as

$$x(t) = A\sin(\omega t + \phi_x) \qquad (3)$$

where ω is the angular frequency, A is the maximum displacement (amplitude) and ϕ is the initial phase of the displacement. By comparing equations (1) and (2), for a simple pendulum, angular frequency and hence the time period of the oscillation T, are given by

$$\omega = \sqrt{\frac{g}{L}}; \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad (4)$$

The period is precisely independent of *m*, which reflects the fact that the acceleration of gravity is independent of *m*. In this experiment, we will investigate the dependence of the period of the oscillation on *L* and *g*. By changing the length of the string, *L* is varied. How can we vary acceleration of gravity?

In order to study properties of SHM we will use a motion detector to measure the displacement, velocity and acceleration of a pendulum with respect to times. Velocity and acceleration are given by

$$v(t) = A\omega \sin(\omega t + \phi_v); \qquad a(t) = A\omega^2 \sin(\omega t + \phi_a) \qquad (5)$$

where $A\omega$ and $A\omega^2$ are maximum velocity (V_{max}) and maximum acceleration (a_{max}) respectively, and ϕ_v and ϕ_a are phases of velocity and acceleration respectively.

The maximum values of the displacement, velocity and acceleration are related as

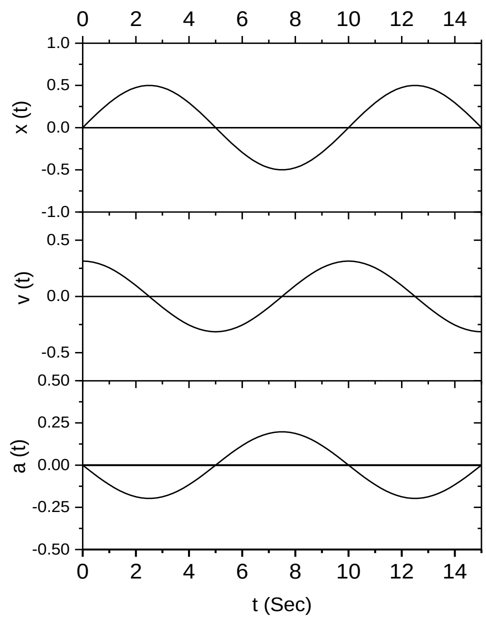
$$\frac{maximum\ velocity\ (v_{max})}{maximum\ displacement\ (A)} = \frac{A\omega}{A} = \omega$$

$$\frac{maximum\ acceleration\ (a_{max})}{maximum\ displacement\ (A)} = \frac{A\omega^2}{A} = \omega^2 \qquad (6)$$

and phases are related as

$$\begin{aligned} \phi_v - \phi_x &= \pi/2 \quad (90^o) \\ \phi_a - \phi_x &= \pi \qquad (180^o) \end{aligned} \qquad (7)$$

The plot on the right shows an example of variation of displacement, velocity and acceleration with respect to time. (The grapes are plotted for A = 0.5 unit, T = 10 s, and $\phi_x = 0$)



Apparatus

Pendulum bob with string, support stand with clamp, stopwatch, meter stick, Vernier caliper, air table with blower, puck with string, motion sensor, Vernier LabQuest interface device and a computer with Logger Pro software.

Description of Apparatus

In order to investigate the dependence of time period of the simple pendulum on the length, you are going to change the length of the string and find the time periods for different length. For dependence of the period on *g*, we won't travel to the moon, where the acceleration of gravity is different, but we will use a frictionless air table as shown in Figure 2. A puck with a string will be used as a simple pendulum and placed on the air table tilted at an angle, θ, with the horizontal as shown in Figure 2. The air flowing out from the tiny holes on the surface of the air table makes it 'frictionless' for a puck.

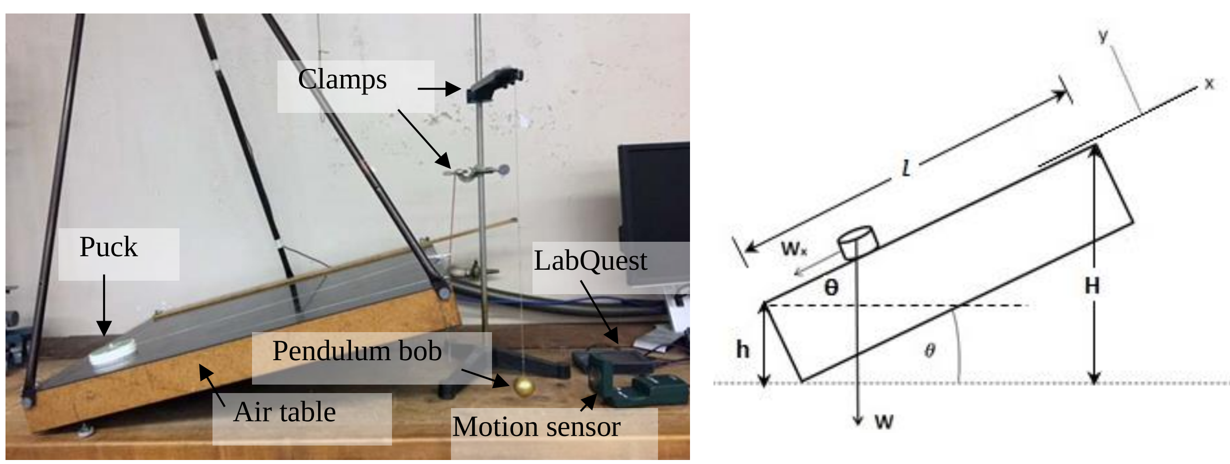


Figure 2: (a) Experimental set up and (b) schematic geometry for the inclined air table.

Figure 2b shows a schematic for the trick to achieve variable g. In this arrangement y-component of the weight is balanced by the normal force from the table, and x-component, $W_x = W \sin \theta$, causes the oscillation of the pendulum. Therefore, we can modify the equation for time period given in Eq. (4) as

$$T = 2\pi\sqrt{\frac{L}{g_{eff}}}, \qquad (7)$$

where g_{eff} is 'effective g' and is given by $g_{eff} = g \sin \theta$. By tilting the air table we can change g_{eff}. The value of $\sin \theta$ can easily be found from the geometry of the air table shown in Figure 2b in terms of H, h and l as

$$\sin\theta = \frac{H-h}{l} \qquad (9)$$

In order to measure the variation of displacement, velocity and acceleration of the pendulum, you will use a motion detector along with a Vernier interface device as in the previous mechanics experiments.

Procedure

Part I. Dependence of time period on the length of the pendulum

1. Measure the mass and the diameter of the bob using a scale and a Vernier caliper respectively. Record them in the data sheet.
2. Attach a clamp to the support stand. Set the string of the pendulum to the clamp and adjust the length of the string to 0.30 m. (You need to add radius of the bob to get length of the pendulum.)
3. Adjust the height of the stand if necessary. Now, pull the bob for a small angle, α (< 10^o) with vertical and release it. Try to keep the pendulum oscillating on a plane. Repeat if necessary. Also, the stand should be kept stable.
4. Once you know how to make nice oscillation, make your stopwatch ready to measure the time. Before starting the stopwatch, you may skip first few cycles. Then measure the time for 10 complete cycles and record in Table 1. Why do we need many oscillations to find the time period?
5. Now, adjust the lengths of the string and repeat the previous step by changing the lengths of the string: 0.4, 0.5, 0.6, 0.8, 1.0, 1.2, and 1.4 m. Record your measurements in the data sheet.

Part II. Dependence of time period on the effective *g*

For this part of experiment, you are going to oscillate a puck attached to a string on the inclined frictionless air table.

1. Attach the puck string to the metal string at the boarder of the air table. The length of the puck string should be about 0.7 m and the puck should not touch the string at the lower edge of the table. Measure the actual length of the string you are using, diameter of the puck and length of the air table. Record your measurements in the data sheet.
2. Hook a nylon cord loop to the centered leg under the side of the air table and a right angle clump of the support stand. The cord will hold the air table and you can adjust the position of the clamp to change the inclination.

3. Just to have idea of angle of inclination and practice the oscillation, set the table to an arbitrary inclined position. In order to find the angle of inclination, measure the heights of the lower and upper edges of the table (see Figure 1b). Calculate the angle and have an idea of inclination angle versus height of the table edge needed to lift. Now, turn on the air blower. The puck should be floating on the air table. Pull the puck about an angle less than 10^o and release it. After a few oscillations, measure the time for 10 oscillations. **Turn off the blower**.
4. You are going to perform this experiment with different angles of inclinations around 10^o, 20^o, 30^o, 40^o and 60^o. Set the inclination angle by changing the position of the right angle clamp on the support stand and record your data in Table 2. Measure the time of 10 oscillations for each case and record in the table.

Part III. Position, velocity and acceleration of the pendulum with respect to time

In this part of the experiment, you are going to use the pendulum bob again to study how the position, velocity and acceleration of the pendulum bob vary with respect to time.

1. Attach a pendulum bob with string to the clamp on the support stand. Adjust the length of the pendulum to about 30 cm. Place a motion detector straight in front of the motion and about 50 cm away from the pendulum bob. The motion detector should be connected to the LabQuest interface device and then to the computer.
2. Open Logger Pro in the computer. You should have empty graphs of position velocity and acceleration versus time. Pull the bob and release as you did in part I. Note the bob should oscillate along the beam of the motion sensor. After a few oscillations, collect the data. If necessary, make some adjustment and repeat it. When the graphs look like sinusoidal, save the data and proceed to analyze the graphs.
3. Set the appropriate scale for the axes of the graph.
4. Highlight a portion of the graph of position versus time. Click ‘Analyze’ on menu bar and select ‘Curve Fit’. Choose ‘Sine’ and click ‘Try Fit’. You will see a fitted curve on the data. The fitted curve should coincide with the graph. If the fitted curve and the graph do not match well, you have to repeat it. Click ‘Done’. A fitted curve and a small box with coefficients will appear. The coefficients A, B, and C represent amplitude, angular frequency and phase of the sine curve.

   Repeat this process for the velocity and acceleration graphs. Write down the coefficients in your data sheet. Print the graphs and include in your lab report.
5. Now, change the length of the pendulum string to about half (~15 cm) and adjust the height of the pendulum clamp so that the bob is in front of the motion detector again. Repeat steps 2 – 4.

Computations

Length of the pendulum is the distance from the pivot to the center of the oscillating object. So, add the radius of bob and puck for all trials to get the length of pendulum in the tables.

Complete the Table 1 using the measured values from Part 1.

From the data in Table 1, plot the graph of $\mathbf{T^2}$ versus **L**, and find the slope. Calculate the experimental value of acceleration due to gravity and compare with standard value of g.

Calculate g_{eff} in Table 2.

Plot the graph T^2 versus $1/g_{eff}$. Determine the slope of the graph. Determine the theoretical value of the slope from equation (7) and compare with the value from the graph.

Complete Table 4 using the data in Table 3. Calculate the time period from the coefficient B. Compare the results with theoretical values from the equation (4).

From the maximum values of the position, velocity and acceleration graphs (fitting coefficients, A), find their ratio and compare if the ratios are valid as in equation (6). The coefficient C from the fitting gives you the phase of the sinusoidal function. Calculate the phase differences between the velocity and position graphs, acceleration and position graphs. Are they close to the value given in the equation (6)? What is the meaning of having phase difference of $\pi/2$ and π?

Questions

1. Taking reference from the graphs of position, velocity, and acceleration of the pendulum, at what points of its path does it reach maximum velocity? At what points of its path does it reach maximum acceleration? Explain with a diagram of swinging bob.
2. Can we consider the pendulum in a pendulum wall clock a simple pendulum? Do you think the time period in the clock is independent of the mass of the pendulum?
3. Suppose the air table is placed horizontal and a spring the attached to the puck instead of the string. What would be the motion of the puck is it is pulled horizontally and released?
4. If you are given a simple pendulum and a standard stopwatch, what are the physical parameters you can measure using them?

Data Sheet

Date experiment performed:

Name of the group members:

I. Dependence of time period on the length of pendulum

Mass of the bob = _________ kg Diameter of the bob = ___________ m

Table 1.

Trial	Length of string (m)	Length of Pendulum, L (m)	Time, t, for 10 oscillations (sec)	Time Period $T = t/10$ (sec)	T^2 (sec^2)
1					
2					
3					
4					
5					
6					
7					
8					

Slope of the graph of T^2 *vs* L =

$g_{exp} = 4\pi^2 / Slope$ = % error =

II. Dependence of time period on effective *g*

Diameter of puck = ___________ m Length of string = __________ m

Length of the pendulum, L = _________ m Length of air table, l = _________ cm

Table 2.

Trial	Height of lower edge of the table, h (cm)	Height of upper edge of the table, H (cm)	$g_{eff} = g \sin\theta$ = g.(H-h)/l (m/sec^2)	$1/g_{eff}$ (sec^2/m)	Time for 10 oscillations, t (sec)	Time Period $T = t/10$ (sec)	T^2 (sec^2)
1							
2							
3							
4							
5							

Slope of the graph T^2 *vs.* $1/g_{eff}$. = ____________ m

Theoretical value of the slope ($4.\pi^2 L$) = __________ m % error =

III. Position, velocity and acceleration of the pendulum bob with respect to time

Table 3.

Trial	Length of Pendulum, L (m)	Position (x) vs t graph			Velocity (v) vs t graph			Acceleration (a) vs t		
		A_x (m)	B_x (sec^{-1})	$\boldsymbol{C_x}$ (rad)	$\boldsymbol{A_v}$ (m)	B_v (sec^{-1})	C_v (rad)	A_a (m)	B_a (sec^{-1})	C_a (rad)
1										
2										

Table 4.

	ω	Amplitudes		Phases		Time Period	
		A_v/A_x	A_a/A_x	$C_v - C_x$	$C_a - C_x$	$T = 2\pi / \omega$ (sec)	T_{theory} (sec)
Trial 1							
Trail 2							