CONSERVATION OF ANGULAR MOMENTUM USING AIR MOUNTED DISKS

Purpose

a. To study the conservation of angular momentum using two air mounted disks.

b. To study the conservation of angular momentum in a rolling ball into an air mounted disk.

Theory

Part I: Inelastic collision of two disks

Conservation of angular momentum states that in absence of external torque, angular momentum ($\boldsymbol{L} = I\,\boldsymbol{\omega}$) of a system is conserved. In an event of collision without external torque involved, angular momentum before collision (L_i) = angular momentum after collision (L_f)

$$I_i \omega_i = I_f \omega_f \tag{1}$$

We will use a Rotational Dynamics apparatus that has two air mounted disks to perform experiment on collision of rotating disks and study this principle. The moment of inertia, I, of a circular disk about the perpendicular axis through its center can be calculated by

$$I = \tfrac{1}{2} M R^2 \tag{2}$$

where M and R are the mass and radius of the circular disk respectively.

The apparatus is designed to have two disks rotate freely on a cushion of air. We will allow the rotating disks to collide and stick together after the collision. Thus, it is an inelastic collision. By measuring the angular velocities (ω) we can calculate the total angular momentum of the system before and after the collision.

Part II: Inelastic collision with a rolling steel ball and an air mounted disk

In the second part of the experiment, a rolling steel ball is launched horizontally into an air mounted disk which is initially at rest. Consider a ball of mass, m is moving with speed, v. The linear momentum of the ball, $p = mv$. If b is the perpendicular distance from the center of the rotation to the line of projection (called the impact parameter in this experiment), the angular momentum of the steel ball is given by

$$L_i = mv.b \tag{3}$$

The disk along with a ball catcher is kept initially at rest. Thus, the above equation gives the initial angular momentum of the system. Once the ball lands onto the disk the ball and the disk both rotate with same angular velocity (ω). The final angular momentum of the system after collision can be written as

$$L_f = (I_{dc} + mb^2)\omega \tag{4}$$

where I_{dc} is the moment of inertia of the disk with the catcher and ω is angular velocity after collision. From the conservation of angular momentum, equations (3) and (4) give final angular velocity as

$$\omega = \frac{mvb}{I_{dc} + mb^2} \tag{5}$$

Apparatus

Rotational dynamics apparatus, Ball ramp, catcher, and ball, weights (5, 10, 20 grams), Triple beam balance, White paper, carbon paper, and mask tape, Vernier caliper, and meter stick.

Description of Rotational Dynamics Apparatus

The apparatus, see Fig. 1, consists of a base plate mounted with a spindle, display housing with a switch to monitor top or bottom disk, an air bearing pulley, and three valve pin holes. The top and bottom disks are placed over the spindle. The apparatus allows frictionless rotation of the disks by compressed air delivered between the disks and the spindle. **Do not rotate the disks against each other unless the compressed air is delivered to the system.**

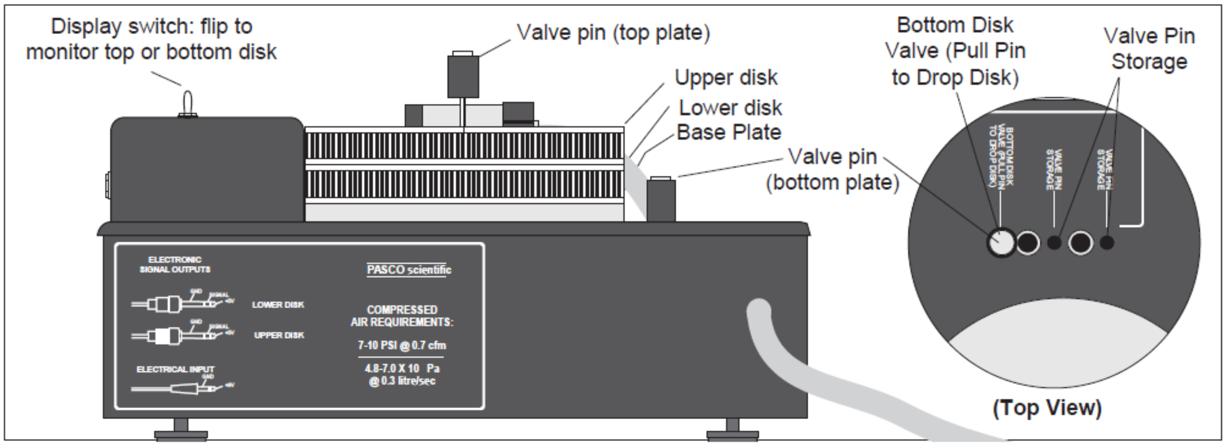

Figure 1. Rotational Dynamics Apparatus

Both disks can spin independently or together, or the upper disk can spin while the lower disk does not. These options are controlled using two valve pins. If one places a valve pin in the bottom disk valve, located next to the valve pin storage, the compressed air goes between this disk and the base plate, and reduces friction. To adjust independent rotation of the disks, one should place the hollow thumbscrew with a valve pin at the center of the upper disk. When the pin is removed, air goes through the screw hole and disks rotate together.

The disks are marked by 200 black bars around the rim. The digital display unit has optical detectors that count the number of black bar pass by while rotating and displays the number of bars per second, n. The switch on the top of the display housing allows you to select the reading from the upper or the lower disk. The measurement is made and the display is updated every 2 seconds. Thus, when you flip the switch, skip the first reading or wait for a full two seconds before recording data. Since the display shows the number of black bars, n**,** that passes the optical detector each second, the angular velocity in radians/ sec can be calculated as

$$\omega = \frac{2\pi n}{200} \tag{6}$$

The timing cycle for the optical readers lasts two seconds. During the first second of each cycle, the optical detector counts the black bars that pass by. During the first few milliseconds of the second half of the cycle, this count is displayed. Then a dead time follows until the full two-second cycle is complete. The cycle begins again. That means, when an angular acceleration is measured, the difference of *two successive counts*, Δn_{av} should be divided by 2 seconds.

$$\alpha = \frac{\Delta\omega}{2} = \frac{\pi \Delta n_{av}}{200} \tag{7}$$

Procedure

Part I: Inelastic collision of two disks

Getting started: Two disks should have been set on the spindle (Fig. 2). It is very important to level the air disks and the base of the apparatus. **Don't change the position of the leveled apparatus and don't remove the disks.** Adjust the pressure of the compressed air to about 9 psi. Screw a short hollow thumb screw at the center of the upper disk. Insert a valve pins so that the two disks of the apparatus are free to rotate independently. Spin the disks. They should rotate smoothly and independently. When the valve pin removed from the top disk, both disks should stick and rotate together. If the disks continue rotating separately, lower the air pressure.

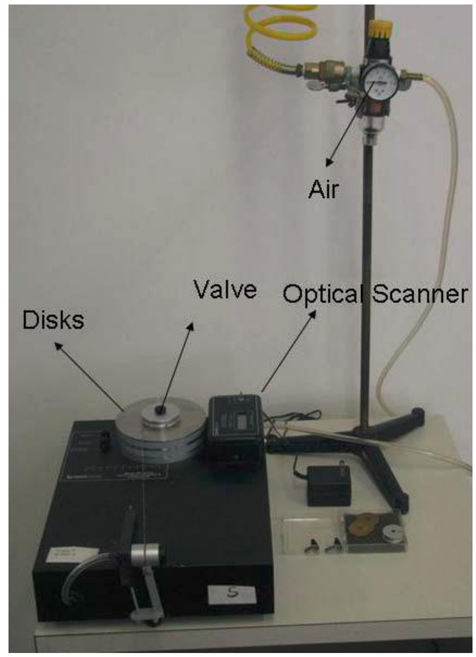

Figure 2. Set up for part I.

1. Record the diameter of the disks and the masses of the disks printed on the apparatus in Table 1 to calculate the moment of inertia of the rotating disks.
2. Set the display switch in a position depending upon which disk is rotating. You can start with the lower disk at rest and the upper disk in rotation. Spin the disk so that the display reads approximately 300-400 counts/sec. Record the number from the digital display in Table 2. Now, remove the pin from the upper disk so that the two disks rotate together. Record the number from the digital display for further calculation and analysis.
3. Repeat the trial for upper disk at rest and lower disk in rotation.
4. Repeat the trial for both disks rotating in the same direction but with different ω.
5. Repeat the trial for both disks rotating in opposite directions.

Note: After completing the experiment reduce the air pressure to zero.

Part II: Inelastic collision with a rolling steel ball and an air mounted disk

In this part of the experiment, you are going to roll a steel ball on a ramp and launch the ball horizontally into the upper disk.

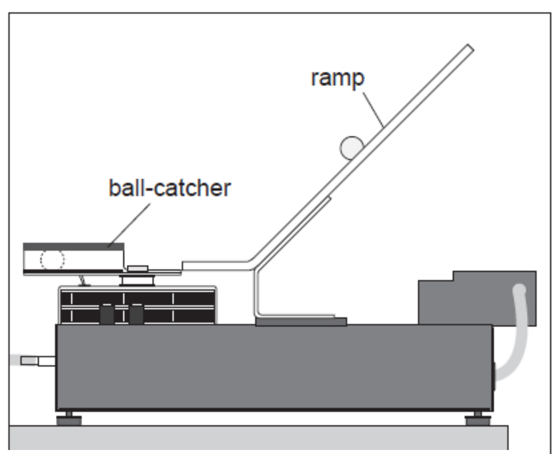

Figure 3. Set up for inelastic collision of a disk with a rolling ball.

1. Set up the small torque pulley with thread and ball catcher on the upper disk with a long thumb screw. Keep both valve pins in the valve storage so that the bottom disk rests firmly on the base. Switch the display to read from the upper disk.
2. Before launching, hold the disk stationary and position the launcher so that the ball will be caught at a radius of 4 cm from the center of rotation of the disk. This radius is called the *impact parameter*, *b*.
3. Release the ball from the top of the ramp and permit the catcher to catch the ball. ***Quickly move the launcher out of the way***. The disk will begin

rotating. Record the digital display to calculate the angular velocity after collision.

4. Make four measurements at each of the impact parameters: 3, 4, 5, 6, 7 and 8 cm. Record your measurements in Table 4.

Note: reduce the air pressure to zero after performing the experiment.

Determination of the launching speed of the ball

In order to determine the linear momentum of the ball or the initial angular momentum, you have to measure the launching speed of the ball. The launch speed (just before hitting the catcher) depends on the initial height the ball starts rolling. You may follow the following technique to measure it.

1. Place the ramp near the edge of the laboratory table as shown in Figure 4 so that the ball, when launched, goes over the edge of the table and lands on the floor. From the knowledge of the height of the launch point above the floor and the horizontal distance traversed by the ball during its fall, we can determine its initial velocity.
2. To record the horizontal position at which the ball hits the floor, you can tape a piece of white paper on the floor at approximate point of impact, and cover it by a sheet of carbon paper. **Don't tape the carbon paper**. Release the ball from the top of the ramp same as in part II. Repeat this step of releasing the ball for few times from the same starting point and same way. You should see the same number of carbon spots on the white paper. Find an optimum position of the spots from several trials and measure the range *d*, i.e., the distance from the marks on the paper to the point on the floor vertically under to the edge of ramp. Measure the vertical height, *h*. The average speed can be estimated from

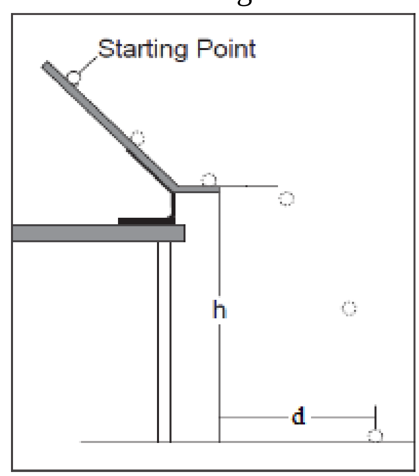

Figure 4. Measuring launching speed of the ball.

$$v = d\sqrt{\frac{g}{2h}} \qquad (8)$$

Determination of Moment of Inertia of the disk with the catcher

In the part II of the experiment, the upper disk is attached with the torque pulley and catcher. Thus the moment of inertia is different from the one you have calculated using equation (2). We can apply Newton's second law to determine the moment of inertia of the disk with catcher (I_{dc}) by attaching a hanging mass to a string and connected to the torque pulley (Fig. 4). The hanging mass applies a constant torque on the air mount disk system. Angular acceleration, α of the disk is given by

$$\alpha = \frac{m'gr}{I_{dc}+m'r^2} \qquad (9)$$

where m' is the mass of the hanging object, r is the radius of the torque pulley (it is not the radius of the disk!). By measuring α, we can determine I_{dc}.

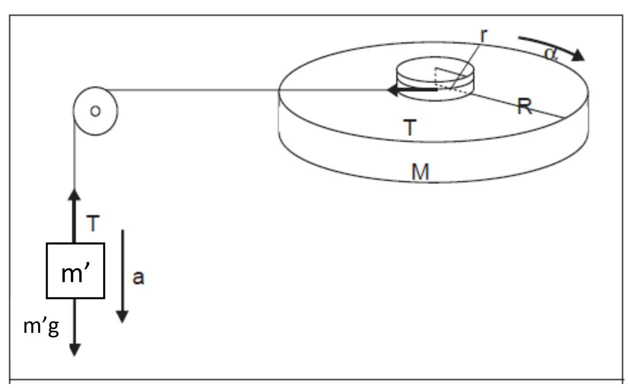

Figure 5. Determining moment of inertia

1. Measure the diameter of torque pulley. Put a thread over the torque pulley on the upper disk along with a ball catcher. Draw the cord over

the air pulley so that the weight holder hangs over the edge of the table.

2. Adjust the air pressure between 5-9 psi. Rotate the disk so as to wind the cord onto the pulley, taking care to leave the weight holder suspended near the edge of the apparatus. You are now poised to begin a measurement by allowing the upper disk to rotate freely. Once it starts rotating, skip the first counter reading and record the successive readings from the display. Determine Δn_{av} and calculate the angular acceleration from Eq. 7.

   Under the circumstance that the mass of the disk $M >> m'$ and the radius of the cylinder $R >> r$, the formula to calculate moment of inertia can be simplified to

$$I_{dc} = \frac{m' g r}{\alpha} \tag{10}$$

3. Repeat this step for three different hanging masses and determine the average I_{dc}.

Computation

From the data in table calculate the moment of inertia of the disks.

Calculate the angular momentum before and after the collision for each trial in Table 2 and compare them to see if your experiment verifies the conservation of angular momentum.

In Table 3, calculate rotational kinetic energy before and after the collision from the information in table 1 and 2.

Determine the launching speed of the ball from equation (8).

Calculate final angular velocity for different impact parameter in Table 4.

Plot a graph of ω vs b. If you do an order of magnitude calculation, you will notice $m\,\boldsymbol{b}^2 << I_{dc}$. So the graph of $\boldsymbol{\omega_{exp}}$ vs. b is essentially a straight line.

Find the slope of the line. What do you get from the slope of the line?

Determine I_{dc} from the slope and compare with your experimental result from Table 5.

Questions:

1. Why is angular momentum conserved in these collisions (Parts I and II)? Note that linear momentum is not conserved in Part II. How then is angular momentum conserved?
2. Derive Equation 9.
3. Is kinetic energy conserved in Part II? If not, calculate the fractional loss of kinetic energy for at least one of the cases you measured.

Data Sheet

Date experiment performed:

Name of the group members:

Table 1. Moment of Inertia of the disks

	Mass, M (kg)	D (m)	R (m)	I (kg.m^2)
Upper disk				I_{upper} =
Lower disk				I_{upper} =

Table 2. Inelastic collision of two disks

Trial	Reading of counter, n			Angular velocity, ω (rad/s)			Angular momentum, L (kg.m^2/s)				%diff.
	Before collision		After	Before		After	Before collision			After	
	Upper	Lower		Upper	Lower		Upper	Lower	Total		
1											
2											
3											
4											

Table 3. Kinetic Energy

Trial	Rotation kinetic energy (**J**)			
	Before collision			After collision
	Upper	Lower	Total	
1				
2				
3				
4				

Table 4. : Inelastic collision of the ball to disk for different impact parameters

Impact parameter, *b* (m)	Counter reading, *n*				n_{av}	ω (rad/s)

Table 5: Determination of the moment of inertia of the disk and catcher

Radius of the spool r = __________ m

Trial	Hanging Mass, m' (kg)	Successive counter readings, *n*					Δn_{av}	α (rad/s^2)	I_{dc} (kg. m^2)	$(I_{dc})_{av}$ (kg. m^2)