ONE-DIMENSIONAL COLLISIONS

Purpose

    a. To verify the law of conservation of linear momentum in one-dimensional collisions.
    b. To study conservation of energy and linear momentum in both elastic and inelastic one-dimensional collisions.
    c. To study impulsive force during the collision.

Theory

Conservation of linear momentum is the most important implications of Newton's law. Linear momentum (p) for a particle of mass, m moving with velocity, v, is defined as $p = mv$. The momentum (p) and velocity (v) are both vectors and have the same direction. For a system of n particles with masses $m_1, m_2, m_3, ..., m_n$, with respective velocities $v_1, v_2, v_3......, v_n$, the linear momentum of the system is the vector sum of the individual momentum, i.e.,

$$p_{sys} = p_1 + p_2 + ... + p_n = m_1 v_1 + m_2 v_2 + ... + m_n v_n. \tag{1}$$

The kinetic energy of a particle of mass m and velocity v is defined as $KE = \frac{1}{2} m v^2$. (Note that KE is a scalar quantity). The kinetic energy of the system of n particles is given by

$$KE_{sys} = KE_1 + KE_2 + ... + KE_n = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + ... + \frac{1}{2} m_n v_n^2. \tag{2}$$

The conservation of linear momentum states that if the net force acting on a system of particles is zero then the linear momentum of the system is conserved (i.e., p_{sys} is constant). In isolated collision problems the net linear momentum before and after the collision must be same

$$p_{sys} \text{ before collision} = p_{sys} \text{ after collision} \tag{3}$$

For the one dimensional collision problem involving two objects as shown in Figure 1, we can write equation (3) as

$$p_{1i} + p_{2i} = p_{1f} + p_{2f} \tag{4}$$

where i, f refer to the initial and final stages. Direction of velocity (or momentum) is taken position when the object is moving to the right as indicated by arrow heads in Figure 1.

Figure 1. One dimensional collision involving two objects.

In our experiments, we will study the collision of two carts (cart-1 and cart-2) moving on a horizontal track. The carts and the track are designed for negligible friction. Since there are no external forces acting on the carts during the collision, p_{sys} must be conserved. Additionally, we will keep one of

the carts (cart-2) at rest before collision for all cases (i.e., $v_{2i} = 0$ hence $p_{2i} = 0$). For this condition, we can simplify equation (4) as

$$m_1 \, \mathbf{v}_{1i} = m_1 \, \mathbf{v}_{1f} + m_2 \, \mathbf{v}_{2f} \tag{5}$$

By measuring the masses and the velocities before and after the collision, we can easily verify the conservation of linear momentum. In this lab, we will investigate two cases.

Part I: Perfectly inelastic collision

In the case of a perfectly inelastic collision, the carts stick together after the collision. Thus, both carts will have the same final velocities, that is $v_{1f} = v_{2f} = v_f$. Using this condition in equation (5), we get

$$v_f = \frac{m_1 v_{1i}}{(m_1 + m_2)}. \tag{6}$$

Note: The KE_{sys} is not conserved in a perfectly inelastic collision.

If the kinetic energy is not conserved where does the energy go?

Part II: Elastic collision

By definition, in an elastic collision, the KE_{sys} is conserved, *i.e.*

KE_{sys} before collision $= KE_{sys}$ after collision

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \tag{7}$$

Solving eqns. (5) and (7) simultaneously, we obtain for elastic collisions,

$$v_{1f} = \frac{(m_1 - m_2)v_{1i}}{(m_1 + m_2)} \quad \text{and} \quad v_{2f} = \frac{2m_1 v_{1i}}{(m_1 + m_2)} \tag{8}$$

Part III: Impulse during the collision

` During the collision each cart exerts force on each other. Let $\mathbf{F}_{21}$ be the force exerted on cart-2 by cart-1 and Δt is the duration of impact. The impulse on the cart-2 is defined by the product; $\mathbf{F}_{21.} \, \Delta t$, which is equal to the change in momentum. Thus,

$$F_{21}.\Delta t = m_2 v_{2f} - m_2 v_{2i} \tag{9}$$

We can write similar expression for cart-1 and derive equation (4). (see question in computation section)

We will attach a force sensor on each cart to measure the force exerted on the carts which is referred to as the *impulsive force*. From the graph of force versus time during the collision, Impulse can be determined from the area under the curve.

Apparatus

Vernier frictionless carts and a track, motion detector, photogate with timer, force sensors, additional mass, LabQuest interface, Computer with "Logger Pro" software, and Scale.

Description of Apparatus

Figure 2 shows the experimental set up in this lab for the case of a *perfectly inelastic collision*. You will use a motion detector to measure the velocities of a cart before and after the collision. The motion detector should be connected to a Vernier LabQuest interface device which is then connected to a computer. Using the Vernier Logger Pro software, you can collect the data for the position and velocity with respect to time. Velcro stickers on the side of the carts make them stick together after the collision.
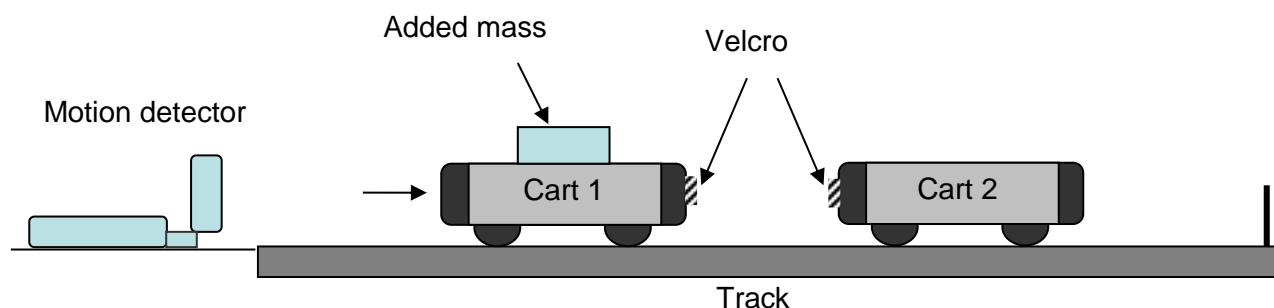

Figure 2: Experimental set up for perfectly inelastic collision.

Figure 4 shows the experimental set up for an *elastic collision*. Elastic bumpers with a Velcro sticker (or embedded magnets) are attached on the front of the carts to make the collision elastic. Both carts will be moving with different velocities after the collision in case of an elastic collision. So you will need an additional sensor to measure the velocity of the cart-2. You will use a photogate to measure the velocity of cart-2 (see Figure 3). The photogate has a timer attached. The photogate starts the timer when an obstacle goes past the photogate and stops the timer when it leaves the photogate. We will install a flag on cart-2 to start and stop the timer. The velocity of the cart is then equal to the length of the flag divided by the time interval.
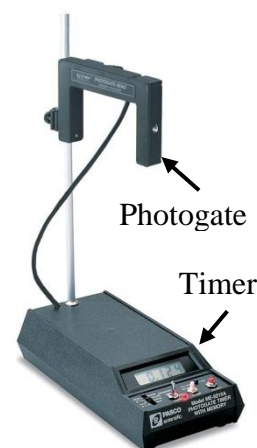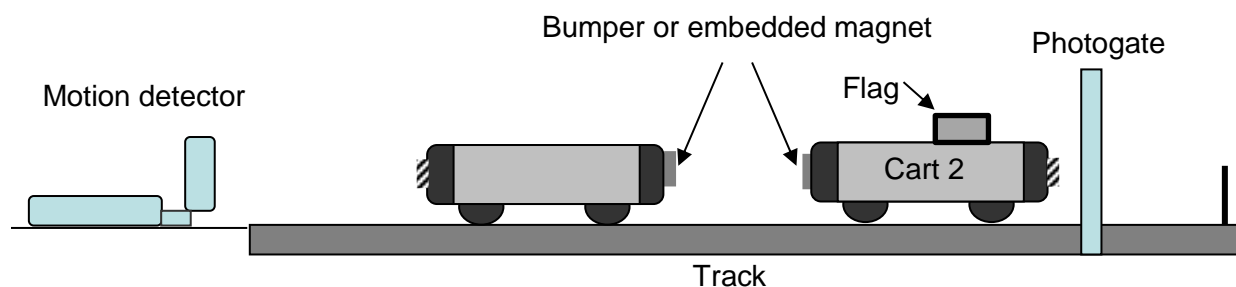

Figure 3: Photogate and timer.

Figure 4: Experimental set up for elastic collision.

Procedure

1. Measure the masses of the carts and additional loads, and record in the data sheet.
2. Attach a motion detector on one side of the track. The motion detector should be about 20 cm away from the edge of the track. The motion detector should be connected to the Vernier LabQuest interface device which then should be connected to the computer. Open '**Logger Pro'** in the computer.
3. Check if the track is leveled. You can easily check if the track is leveled or not by placing a cart on the track. If the cart stays at rest even if you move the cart to another spot, it is leveled. If the track is not leveled, the cart starts moving. You should then level it by adjusting the screws under the track.

Part I: Perfectly inelastic collisions

(For the case of perfectly inelastic collision you do not need to use the photogate.)

4. Place cart-2 near the center of the track and the cart-1 at the edge of the track where the motion detector is attached. Make sure the Velcro sticker on the carts face each other so that the carts stick together after the collision for a perfectly inelastic collision.
5. Now, get ready to make the collision and collect the data. Place you finger on the back of cart-1 and **make sure your hand does not block the beam of the detector and that no one is standing or moving in front of the motion detector.**
6. Click '*Collect'* on the computer. *You should hear regular clicking sound from the motion detector.* Then push the cart-1 by your finger. DO NOT move your hand while the carts are moving. STOP the carts after it hits the bumper at the other edge of the track.

The program will collect the data for the position of the cart with respect to time. It displays the data of position and velocity in the table. It also displays the graphs of position and velocity as a function of time. *You should see a jump in the plot at the moment the carts collide*. Make sure that the velocity is approximately constant before and after the collision. Repeat until you get a nice run. Save the data and graphs for analysis.

7. **Finding the velocities:** You can get the velocities before and after the collision from both the position and velocity versus time graphs.
   a. The position vs. time graph should show two straight lines of different slopes that meet at the collision time. Highlight a region of the graph using the mouse before the collision and click '*Analyze'* in the menu bar and then click '***Linear Fit'***. A box will appear showing the slope of the linear fitted line. ***What do you get from the slope of the position versus time graph?***
   b. On the velocity vs. time graph, highlight the region on the graph before the collision time, as done in part (a). Click '*Analyze'* in the toolbar menu and then click '***Statistics'***. A box will appear showing the average value of the velocity in the selected graph region. Record the velocities in Table 1. Check if the values are close.
8. Repeat steps (7a) and (7b) by highlighting the regions in the graphs after the collision time. Record the values in Table 1.
9. We will now repeat the trial with different masses of the carts. Add a load on cart-1. Cart-1 is heavier than cart-2. Repeat the steps 4 – 8 to measure the velocities before and after the collision.
10. Now, remove the load from the cart-1 and add onto cart-2. In this case, cart-2 is heavier than cart-1. Repeat the measurement again.

You should include graphs of position and velocity versus time from **one of these trials** in your report.

Part II: Elastic collisions

In order to realize an elastic collision, use elastic bumpers (or two magnets embedded) on the front of the carts and place them facing each other. Put a flag on cart-2. Also measure the length of the flag. You are going to use the photogate to measure the velocity of cart-2 after collision. Place the photogate about 25 cm from the edge of the track on the side cart-2 will move.

1. Plug in the photogate power supply. Switch the photogate settings to: mode = "gate", time = "0.1 ms", and memory = "on".
2. Set the appropriate height of the photogate so that cart-2 with flag can pass freely through the gate and the flag also blocks the light of the photogate (when this happens, the red light on the photogate flashes). The timer is started when the front edge of the flag passes through the photogate. The timer is then stopped when the back edge of the flag leaves the photogate. The velocity of cart-2 is, therefore, length of flag divided by the time interval measured by the timer.
3. **Reset the timer of the photogate**. Repeat steps 4 - 10 of Part I (three different trials). Record your measurements for cart-1 in Table 3. For cart-2, record the timer readings in the table to find the velocity of cart 2. The photogate must be "reset" before each trial.

Part III: What happens during the collision?

In the parts I and II you have investigated the linear momentum of a system before and after the collision. What happens during the collision? You should have observed the sudden changes in the velocity during the collision there by changing the linear momentum of the individual carts. In this part of the experiment, you are going to r investigate further by measuring the forces the carts exert on each other during the collision. You will use the force sensors to measure the force exerted on the carts.

Figure 4 shows the schematic of the experimental set up for this part. Cart-1 is loaded with an added mass. Attach a force sensor on each cart on the rod on the carts. **Set the sensors to ±10 N range**. Connect the sensor wires to the LabQuest interface. You may remove cables of other sensors (motion sensor and photogate) from the interface. Open a new file in the Logger Pro program. You should see force measured by the sensors and graphs to display the forces. The force sensor-1 measures the force exerted on cart-1 and similarly by sensor-2. Click at '*Experiment*' on menu bar and select '*Set up sensors*'. It will show all sensors connected to the interface. You should see two force sensors. **Right click on one of the force sensor and reverse the polarity**. *Why do you think the polarity of one force sensor has to be reversed?*

Before collecting the data**,** change the sampling rate to 200 [*Experiment→Data Collection→* Sampling Rate: 200 samples/sec] and zero the sensors by clicking *Zero* **(0)** next to '*Collect*'**.**
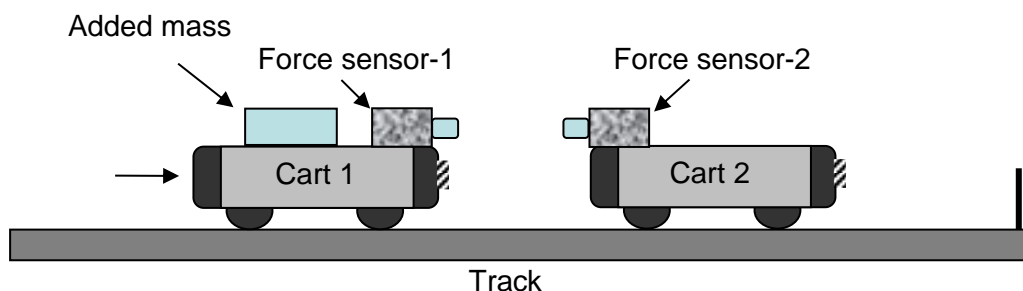

Figure 4: Experimental set up for measuring forces during the collision.

Keep the cart-2 around the middle of the track at rest and the cart-1 at some distance away. The bumpers of the force sensors should face each other as shown in Figure 4. Make sure the wires connected to the sensors do not stretch while they move. You are now ready to see what happens during the collision.

1. Click '***Collect***" in the Logger Pro and gently push cart-1 towards cart-2. Logger Pro will collect the data for the forces exerted on both carts and display them in the graph. You may *Zoom In* the graph for better view. Determine the duration of collision, (Δt) and record in Table 5.

   How does the force exerted on each other vary with time during the collision?

   Compare the magnitude of the forces exerted on each cart at different times during the collision?

2. Highlight the region on the graph while collision occurs for force-1. In the menu bar click, '***Analyze***' and select '***Integrate***'. You will see the area under the plot filled and a small box with the value of area. What is its unit? The value of 'area' is essentially Impulse.

   Repeat it for force-2. Record the values of Impulses in the data sheet and compare.

3. Now, interchange the positions of the carts and repeat the previous steps.

4. What do you expect about the force exerted on each cart if cart-1 is pushing cart-2 and both moving together? Test your prediction.

Computation

From the data you collected in Tables 1 and 3 calculate the theoretical final velocities using Equations (6) and (8) and compare with the experimental values. Calculate the total linear momentum from the experimental results before and after the collision for each trial. Do your measurements indicate conservation of linear momentum? If no, explain the possible sources of error.

Calculate the total kinetic energy before and after the collision for each trial in Tables 2 and 4. Do your measurements indicate conservation of kinetic energy? Explain the possible sources of error.

Calculate the average impulsive forces exerted during the collisions in part III. Also, determine the change in momentum during the collisions. Explain your observation and the conclusion from Part III. Based on your conclusion from the Part III of this lab, derive Equation 4.

Questions

1. What are the differences in your experimental results between the perfectly elastic and inelastic collisions?
2. If the masses of the carts are equal in the elastic collision, what should happen to the carts after the collision? What happened in your experiment? Derive the expression for ratio, $\dfrac{KE_f}{KE_i}$, and compare the theoretical values with your experimental result.
3. When a truck and a car are traveling with same speed make a head-on collision, is the force exerted by the car on the truck equal, greater or lesser than that exerted by the truck on the car?
4. Explain, using linear momentum, why a rifle must recoil in the opposite direction of the bullet when fired?
5. Verify that the units of 'Impulse' and 'Change in momentum' are essentially same.

Data Sheet

Date experiment performed:

Name of the group members:

Perfectly inelastic collision

Mass of empty cart-1 =   ; Mass of empty cart-2 =   ; Mass of the load =

Table 1: Velocity and Momentum

(Note v_{2i} for cart-2 is zero)

Trial #	Cart-1 m_1 (kg)	Cart-2 m_2 (kg)	Before collision (Only Cart-1 moving)				After collision (Carts stocked together)			Theory v_f (m/s)	p_{sys}^f (kg.m/s)	Change in p_{sys} (%)
			v_{1i} (m/s)				Expt. v_f (m/s)					
			Graph-1 (x vs t)	Graph-2 (v vs t)	Average	p_{sys}^i (kg.m/s)	Graph-1 (x vs t)	Graph-2 (v vs t)	Average			
1												
2												
3												

Are the theoretically predicted and experimentally observed values for the final velocity the same?

Is the linear momentum conserved for all trials?

Table 2: Kinetic energy

| | Cart-1 | Cart-2 | Before collision | | | After collision | | |
| | | | Cart-1 | Cart-2 | | Stocked carts | | |
Trial #	m_1 (kg)	m_2 (kg)	*Average* v_{1i} (m/s)	v_{2i} (m/s)	KE_{sys}^{i} (J)	Average Expt. v_f (m/s)	KE_{sys}^{f} (J)	Change in KE_{sys} (%)
1				0				
2				0				
3				0				

Is the kinetic energy conserved in all trials?

Elastic collision

Mass of empty cart-1 =   ;   Mass of empty cart-2 =   ; Mass of the load =

Length of the flag, L =

Table 3: Velocity and Momentum

(Note v_{2i} for cart-2 is zero)

	Cart-1	Cart-2	Before collision				After collision							
			Cart-1				Cart-1					Cart-2		
Trial #	m_1 (kg)	m_2 (kg)	v_{1i} (m/s)			p_{sys}^{i} (kg.m/s)	Expt. v_f (m/s)			Theory v_{1f} (m/s)	Δt (s)	$v_{2f}=$ L/Δt	y_f^{i} (kg.m/s)	Change in p_{sys} (%)
			graph-1 (x vs t)	graph-2 (v vs t)	average		graph-1 (x vs t)	graph-2 (v vs t)	average					
1														
2														
3														

Are the theoretically predicted and experimentally observed values of the final velocity the same?

Is the linear momentum conserved for all trials?

Table 4: Kinetic energy

			Before collision			After collision			
	Cart-1	Cart-2	Cart-1	Cart-2		Cart-1	Cart-2		
Trial #	m_1 (kg)	m_2 (kg)	Average v_{1i} (m/s)	v_{2i} (m/s)	KE_{sys}^{i} (J)	Average Expt. v_{1f} (m/s)	Expt. v_{2f} (m/s)	KE_{sys}^{f} (J)	Change in KE_{sys} (%)
1				0					
2				0					
3				0					

Is the kinetic energy conserved in all trials?

Table 5: Impulse

Trials #	Duration of collision, Δt (s)	Impulse on Cart-1 (N.s)	Impulse on Cart-2 (N.s)	F_{avg} on Cart-1 (N)	F_{avg} on Cart-2 (N)
1					
2					

Based on your impulse measurements, determine the ΔP for Cart-1 and Cart-2 for both trials.