Conservative Force System

This laboratory exercise explores applications of the work-energy theorem using simulations. The lab includes two parts, incorporating conservation of energy when changes in kinetic, gravitational and/or spring potential energy are accounted for without frictional losses.

I. PART I: HOOKE’S LAW AND CONSERVATION OF MECHANICAL ENERGY

A. Hooke’s Law

Many systems in nature, such as biological polymers (e.g. elastin, spider silk, collagen), exhibit the property of returning to normal shape after being compressed or stretched. For example, the aortic valve in your body, largely comprised of elastin, undergoes billions of stretch-strain cycles throughout your lifespan, with little loss in its physical characteristics. Hooke’s law dictates that for short elongation or compression, these systems exhibit the property that the deformation is linearly proportional to the applied force. In part one of this lab we will explore the physics of Hooke’s law and the consequences of energy conservation by studying a mass attached to a vertical spring in the presence of gravity.

When an object of mass m is attached at the lower end of a vertical spring, it elongates and eventually comes to equilibrium. Hooke’s law states that the spring force \( F = k \Delta x \), where \( \Delta x \) is the displacement (elongation) of the spring from its unstretched position \( x_0 \) as shown in Figure 1. In Hooke’s Law the constant ‘k’ is known as the spring constant; a larger spring constant denotes a stiffer spring. At the equilibrium position the spring force is balanced by the weight of the object attached. For an ideal, non-dissipative spring having negligible mass Newton’s law gives us

\[
k \Delta x = mg
\]

In the first part of this lab, we will investigate this relation and determine the spring constant for a spring. In addition we will explore the conservation of mechanical energy for a mass on a spring.

B. Conservation of mechanical energy

In a conservative force system the work done \( (W_c) \) by the force can be expressed as the negative of the change in the potential energy \( (W_c = -\Delta U) \), and is independent of the path. An example of a conservative force is gravity. On the other hand work done in the presence of a nonconservative force (like friction) will depend on the path taken. Potential energy decreases (increases) when a conservative force does positive (negative) work. In the absence of friction, total mechanical energy (kinetic plus potential energy) is always conserved. Below, you will explore the physics of a mass on a spring, and apply the basic principle of conservation of energy to determine the maximum velocity of an object on a vertical spring.

Recall, that the kinetic energy of a mass is given by the expression

\[
KE = \frac{1}{2} mv^2
\]
The spring potential energy, $U_s$, is given by

$$U_s = \frac{1}{2}k(\Delta x)^2 \quad (3)$$

where $\Delta x$ is the displacement of the spring relative to its unstretched position $x_0$ at any time (see Figure 1). Note that the spring potential energy $U_s$ is zero when there is no mass placed upon it and it is not stretched.

For a mass on a vertical spring there is also a gravitational potential energy that needs to be accounted for. This if often measured from some arbitrary position (in the simulation we will take the table height to be $h=0$).

$$U_g = mgh \quad (4)$$

Assuming the mass of the spring may be ignored (in real-world experiments this may be accounted for), the total mechanical energy of the system may be written as the sum of kinetic, gravitational, and spring potential energies.

$$TotalEnergy = KE + U_s + U_g \quad (5)$$

In the absence of friction or air resistance (any dampening) the mechanical total energy of the mass on the spring is conserved,

$$TotalEnergy = \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta x)^2 + mgh = Constant \quad (6)$$
Starting the online software

This laboratory exercise makes use of free web-based simulations. Below we summarize some of the features of the simulation. It is recommended to run the simulation on a computer (or tablet) so that you can see all the buttons. Running the simulation on a small screen like your smart phone may make seeing all the buttons difficult.

Go here: (cut and paste into any browser)
https://phet.colorado.edu/en/simulation/masses-and-springs

and click on the html 5 link as shown in the figure below:

Click on the 'Lab' option:
Measuring a spring constant by applying a constant force to a spring
In this part of the lab we will measure the spring constant ‘k’ which you will make use of in part II of the lab.

Step 1: Adjust the spring constant to any value (make a change so that your data is not replicated throughout your lab section). Do not use the smallest or largest setting-use any value between these values. Do not change this once you have begun the lab. You should not replicate a friend’s value.

Step 2: Turn the dampening off.

Step 3: Place markers for the displacement and mass equilibrium.

Step 4: Place the orange mass on the spring, by picking it up with your mouse and setting it onto the spring. Once the mass is on the spring, pull it down away from the black line so that it oscillates 2 to 3 cm on your screen. Note that the mass will have gravitational potential energy, as it is raised off the table (denoted Height=0) and at the black line. Additionally when the mass is at the black line is has no spring potential energy.

Step 5: You can press the stop button at any time to make measurements.

Step 6: Drag the ruler from the right and align it so that the top is aligned with the blue line. The distance from the blue line to the black line represents the length the spring is stretched, due to the weight of the orange mass you have placed. Note, that you can move the ruler around to make measurements easier.
Step 7: Fill out Table 1 (below) by adjusting the mass from 50g to 300g adding 50g each step. You can always stop the spring from shaking by pressing the stop button as in step 5. Using the ruler make measurements of the displacement $\Delta x$ of the spring (the distance from the blue to the black line).

<table>
<thead>
<tr>
<th>mass (kg)</th>
<th>weight (N)</th>
<th>$\Delta x$ (m)</th>
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<tbody>
<tr>
<td>0.05</td>
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<td>0.30</td>
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</tbody>
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**Computations** For this part of the lab, construct a graph of the applied force ($F=mg$) versus the displacement of the spring ($\Delta x$). Your graph can be drawn in Excel or any other graphing software, or even by hand (include a picture in your lab report). From the slope of the data, determine the spring constant (k). You will use this value in all your calculations that follow.
Characterizing the total energy of the mass on the spring; making measurements of maximum velocity

**Instructions**

Step 1: Take the orange weight off the spring.

Step 2: Toggle the mass to 100g.

Step 3: Toggle the option to view the simulation ‘slow’ so that you can visualize the motion of the mass on the spring.

Step 4: Carefully place the orange mass again on the spring but do so that the top of the mass is exactly at the blue dotted line (the equilibrium line). This will ensure that it does not run off the screen and we can make measurements of some physically relevant parameters.

Step 5: Toggle the option to observe the velocity vector on the right. Notice there are points in the motion where the velocity is zero, or maximum. The velocity vector continues to change throughout the motion of the mass as it oscillates.

**Computations: Determining the total energy when v=0**

Use the pause button (on the bottom right corner) to stop the simulation where the velocity is zero and determine the total energy of the system. It may be helpful to have the simulation show you the velocity vector by toggling it on (click on velocity on the lower left hand corner). With some care, you should be able to pause the simulation when v=0, and kinetic energy=0. For the case v=0, we can determine the total energy by determining the gravitational and spring potential energies.

\[ U_g = mgh = \text{_______________} \]
Computations: Determining the maximum velocity

Use the pause button to pause the simulation where the velocity is maximum. Because energy is conserved, the total energy of the system does not change, therefore you should be able to determine the velocity from your results above. Note that the displacement ($\Delta x'$) and the height ($h'$) will be different than what you used above for the case when $v=0$.

\[
\text{Total Energy} = \frac{1}{2}mv_{\text{max}}^2 + \frac{1}{2}k(\Delta x')^2 + mgh'
\]  

(9)

Lets break down each term above:

Total Energy= _____________ (from above)

\[
\frac{1}{2}k(\Delta x')^2 = _____________
\]  

(10)

\[
mgh' = _____________
\]  

(11)

Do the algebra and solve for $v_{\text{max}}$ (obtain a numerical value)

\[
v_{\text{max}} = _____________
\]  

(12)

If you did this part of the lab correctly you can verify that the maximum velocity may also be determined from an expression we will see later in the semester when we get to simple harmonic motion. Lets do that to check you did it correctly:

\[
v_{\text{max}} = A\sqrt{k/m} = _____________
\]  

(13)

In the above expression ‘A’ is the maximum stretch of the spring measured from the equilibrium position (the black line). For an accurate measurement of A, be sure to measure from the center of the mass (at maximum extension) to the black line.

An alternative method to check your answer is to confirm that the sum of all energies $\frac{1}{2}mv_{\text{max}}^2 + \frac{1}{2}k(\Delta x')^2 + mgh'$ are equal to the total energy you determined above when $v=0$. Note that $h'$ and $x'$ refer to the height of the mass from the table and the mass’s distance from equilibrium respectively when $v=v_{\text{max}}$. 

\[
U_s = \frac{1}{2}k(\Delta x)^2 = _____________
\]  

(8)

Total Energy = _____________
II. PART II: WORK-ENERGY THEOREM: CONNECTED MASSES WITHOUT FRICTION

In this part of the lab you will study changes in kinetic energy and gravitational potential energy for a system of two connected masses in the absence of friction. Again, the work-energy theorem gives us the following (note that we ignored the spring term, as no springs are present in this system).

\[ 0 = \Delta U_g + \Delta KE \] \hspace{1cm} (14)

The terms \( U_g \) and \( KE \) are given in equations 2 and 4. Note that energy is a scaler, and the above expression can be easily modified to include two (or more) objects as a system. That is, for two objects we would have

\[ 0 = \Delta U_{g1} + \Delta KE_1 + \Delta U_{g2} + \Delta KE_2 \] \hspace{1cm} (15)

In the above expressions the subscripts 1 and 2 refer to the two masses. We will use the Atwood machine simulation to observe the effect of ‘g’ on Mars, and verify energy conservation.

Simulations
Go to this page:

http://www.thephysicsaviary.com/Physics/Programs/Labs/AtwoodLab/

Instructions
Step 1: Click on the ‘begin’ button to start the simulation.
Step 2: You can also toggle the location where the simulation is run, by clicking on the upper right hand corner (Click on ‘Earth’ to change the planet and select Mars).

Step 3: You should vary the masses (you can and should change them from what they are set to) and observe the position and velocity of mass 2 which is above the detector.

Step 4: Study the graphs of velocity and position which appear below the apparatus (you have to scroll down).
Computations

1. Solve the energy conservation equation (15) above for ‘g’. Obtain an algebraic solution for ‘g’ in terms of \( m_1, m_2, v_{f1}, v_{f2}, v_{i1}, v_{i2}, \Delta h_1, \text{ and } \Delta h_2 \). For full credit you should show all work in the derivation. Note, as the masses are connected, they have the same velocity magnitude and opposite displacements (mass 2 goes up, mass 1 goes down). Use two arbitrary points on the graph (not the end points) and your answer from the algebra you did above to determine ‘g’. Use the graphs which appear on the bottom of the screen (scroll down). Compare your answer with the known values of ‘g’ on Mars (3.7 m/s\(^2\)).

2. Verify that the total energy of the system of two masses is conserved by computing the sum of potential and kinetic energy of the two masses at any two times using data from the graphs. Use any two points in the data (but not the beginning and end). That is, to be clear, use any two points other than the beginning and end of the data shown in the graphs. Show all work for full credit.
III. QUESTIONS TO BE ANSWERED IN YOUR LAB REPORT

1. You notice that in the first part of the lab, with the mass moving up and down that the velocity is continuously changing. May you use the kinematic equations to determine the position or any other parameters (e.g. velocity or acceleration). Why or why not? Explain your answer for full credit.

2. How well does the spring obey Hooke's law in the simulation? Explain.

3. This question refers to part one of the lab (the mass on the spring). How would a graph of the total mechanical energy versus time look if a non-conservative force, such as air resistance, is included?

4. This question refers to part one of the lab (the mass on the spring). For the mass oscillating on the spring, sketch a graph (by hand) of the kinetic energy versus time. On your graph where are the point(s) where the spring potential energy is zero? Mark these points clearly so that your instructor can easily grade this. You do not need to put numbers on the axis, but sketch the general shape of the curve as a function of time.

5. This question refers to part one of the lab (the mass on the spring). Sketch a graph of the spring potential energy versus time. On your graph where are the point(s) where the kinetic energy is zero? Mark these points clearly so that your instructor can easily grade this. You do not need to put numbers on the axis, but sketch the general shape of the curve as a function of time.

6. Although this lab did not include any experiments, lets imagine that you actually did the experiments instead of the simulations. In part 1 with the mass on the spring, give an example of an instrumental, systematic, and human error that you think you might encounter. Note there are three examples you should give and discuss for full credit.