Measurements and Acceleration due to gravity

Purpose

1. To express the fall time for a free fall experiment as best estimate & uncertainty

2. To measure the acceleration due to gravity near Earth surface.

Introduction

Velocity is defined as the time rate of change of position: $\boldsymbol{v} = \frac{\Delta x}{\Delta t}$. The unit for velocity is meters per second: m/s. If an object is at rest then its velocity = 0 m/s. If the position of the object is changing at a constant time rate and motion is in a fixed direction, for example 2 meters every second, in north direction, then the velocity is said to be constant.

If the velocity is changing with time, then we say that the object is accelerating. The acceleration '$\boldsymbol{a}$' is defined as the time rate of change of velocity: $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$. If the object is speeding up (or slowing down) at a constant rate of change of velocity, for example gaining 3 m/s every second, then the acceleration is constant. The unit for acceleration is the unit of velocity per second; that is m/s every second or m/s/s which we can write as m/s^2.

An object that is under the effect of the force of gravity only is said to be in a 'free fall'. You can notice that for such an object, if we look at it as it is falling down, its speed increases as it falls. Therefore it has acceleration, and for that case of a free falling object the acceleration is found to be constant. It is called the acceleration due to gravity, and its magnitude is given the symbol 'g'.

In this experiment we will do some statistical analysis, then we will apply this analysis to measuring g.

If we set a sphere at a given height above the ground h, and release the sphere from rest, the sphere as mentioned above will accelerate towards the ground with acceleration g. If the height is fixed, then the time of fall should be the same for different trials.
If we try this experiment, we will find that the time is not exactly fixed, but shows a little spread around an average value $t_{average}$:

$$t_{average} = \frac{t_1+t_2+t_3+\cdots+t_N}{N} \qquad (1),$$

where N is the number of trials.

The degree of spread of the values of t from the average t is measured by the standard deviation:

$$\sigma = \sqrt{\frac{\sum_{k=1}^{N}(t_k - t_{average})^2}{N-1}} \qquad (2).$$

In physics we refer to the standard deviation as the uncertainty in the measurement Δt. And the experimental measurement of any variable (here t), is expressed as:

Best estimate ± Uncertainty.

For our case here, the experimental measurement of t is expressed as:

$$t_{average} \pm \sigma_t \qquad (3) \qquad \text{or} \qquad t_{average} \pm \Delta t \qquad (4),$$

where σ_t & Δt are equal. They are the standard deviation and uncertainty in measurement of t respectively.

The *percentage error* between a measured value, $A_{measured}$ and an accepted value is calculated from:

$$\% \; Error = \left| \frac{A_{measured} - A_{accepted}}{A_{accepted}} \right| \times 100\% \qquad (5)$$

running the experiment

The data sheet is on page 4

Part 1: Measuring time of fall, t_{fall}

1) Open the simulator https://www.thephysicsaviary.com/Physics/Programs/Labs/AccelerationOnPlanetLab/

Click begin. Click Ball, and keep & record in the data sheet (on page 4) all the default settings of ball Radius= …….cm, ball mass =…… grams, & ball material: ……

Click Planet, Record, in the data sheet, the Radius of planet, R_{planet} =………………..m. Record the default mass of planet Mass planet, M_{planet} =…………………………kg.

Then click Zoom in. Click Ruler, and move the ball to the 4 m (or close to 4m) height on the ruler, by clicking the ball repeatedly you can set the ball's height. Record the ball height from the lower edge of the ball, $h_{initial \; of \; ball}$ = …………m. Calculate the acceleration due to gravity, for that planet as $g = 6.67 \times 10^{-11} \times \frac{M_{planet}}{R^2_{planet}} \; m/s^2$. (This eqn. will be derived in the gravity chapter of your text book). Does the radius, mass and material of the **ball** affect the calculation of g? Copy all the values above for the planet and g calculated for the planet and your initial height for the ball in the data sheet. Show your calculation of g of the planet in the data sheet.

2) Now click Drop, after the ball totally drops click the graph (to see an expanded view).

Find the time for this first trial of fall $t_{1 \; fall}$ from the graph. The time of fall is the time $t_{1 \; start}$ (on the x-axis) from which the v (y-axis) starts to change from zero till the last small **red circle data point** on the graph, where the corresponding t is $t_{1 \; final}$. See figure 1. (The subscript 1 is to indicate the trial number, k, here $k = 1$).

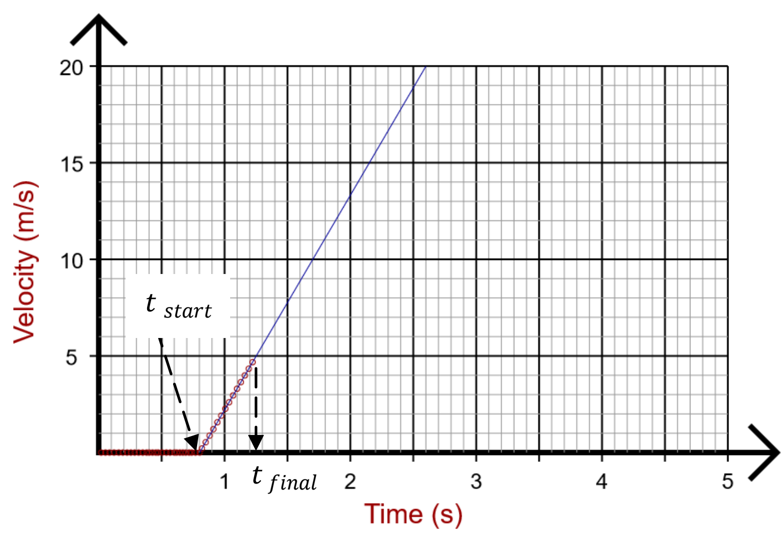

Figure 1: t start & t final shown on the graph

$t_{fall} = t_{final} - t_{start}$.

Record $t_{1 \; start}$ and $t_{1 \; final}$ in table 1 and calculate $t_{1 \; fall}$ and write it in the table as, $t_{1 \; fall}$.

3) Click the graph again to exit the expanded view of the graph, then click Reset.

Click Drop: this is your second trial.

Repeat the steps in 2 to record $t_{2 \; start}$ and $t_{2 \; final}$ and calculate $t_{2 \; fall}$. Record the values in table 1, as your trial 2.

4) Repeat step 3 for 5 more trials (a total of 7 trials), notice that for each trial you have to **reset** before dropping the ball.

5) Calculate $t_{fall\ average}$ using eqn. 1 and the uncertainty in t, Δt using eqn. 2 for the t_{fall} from the 4 m height (or the height you have set to a value close to 4 m in step 1). Write your expression for $t_{fall\ measured}$ as $t_{fall\ average} \pm \Delta t$.

Part 2: Measuring g, the magnitude of the acceleration due to gravity of the Earth

1) If an object is falling freely from rest, the change in vertical position $d = \frac{1}{2}\, \boldsymbol{a}\, t^2$, where $\boldsymbol{a}$ is the acceleration,

here $\boldsymbol{a} = -\, g =$ - 9.81 m/s^2 and t is the time of fall, t_{fall}, from the start of the fall till t_{final}, where y reaches y_{final}.

Open the simulator
https://iwant2study.org/lookangejss/02_newtonianmechanics_2kinematics/ejss_model_freefall01/freefall01_Simulation.xhtml

keep the default settings (free fall, world). Notice we are now using free fall for a ball near the surface of the Earth. g for earth ≃ 9.81 m/s^2 and its direction is downwards.

2) Drag the ball to so that the center of the ball (green point) is at a position of – 20 m.
Click play. As the ball falls completely (notice the simulator measurements are for the final state **right before** the ball touches the ground).
The time of fall and final position are displayed in the yellow box at the lower right of the simulator.
Record t fall in table 2 and calculate the change in position d= final position – initial position= y_{final}- $y_{initial}$, and record in table 2, in the data sheet.
Notice that the final position, y_{final} is always -102.99 m.
Calculate t^2 and record in table 2.

3) Reset the simulator using the reset button .

4) Repeat steps 2 and 3 for initial positions of – 30 m, - 40 m, - 50 m, - 60 m, and -70 m. As mentioned above, the final position, y_{final} is always -102.99 m.

5) For each d and t^2, calculate g from the equation in step 1.

Find $g_{average}$.

Calculate the % error (as given in the introduction in eqn. 5) between your $g_{average}$, and the accepted g = 9.81 m/s^2.

6) If we plot a graph for d (y-axis) versus t^2 (x-axis) what should the slope be?

7) Now plot the graph mentioned in the previous step and find the value of g from the slope. Using eqn. 5 calculate the % error.

The **data sheet** is on the next page

Data sheet

Name: Group: Date experiment performed:

Part 1: Measuring time of fall

Ball radius: Ball mass: Ball material:

$h_{initial\ of\ ball} =$ $m.$ Show your calculation of g_{planet} here:

Mass of planet, M_{planet} (kg)	Radius of planet, R_{planet} (m)	Calculated acceleration due to gravity, g_{planet} (m/s^2)

Does the radius, mass and material of the **ball** affect the calculation of g?

Table 1

Trial #: k	t_k start (s)	t_k final (s)	t_k fall (s)
1			
2			
3			
4			
5			
6			
7			

$t_{fall\ average}$ =

$\Delta t = \sigma_t$ =

$t_{fall\ measured}$ $=$

Part 2: Measuring g, magnitude of the acceleration due to gravity of the Earth

y_{final} = - 102.99 m. (Notice that $\boldsymbol{a} = -g$, as mentioned in step 1 of part 2).

Table 2

$y_{initial}$ (m)	d = y_{final}-$y_{initial}$ (m)	t = t_{fall} (s)	t^2 (s^2)	g calculated (m/s^2)
-20				
-30				
-40				
-50				
-60				
-70				

Show your calculations for g:

$g_{average}$ =

% error=

Slope of graph=

g calculated from slope of graph (show your calculation) =

% error=