

VIDEO ANALYSIS OF ONE- AND TWO- DIMENSIONAL MOTIONS

Purpose

- To study kinematics of one-dimensional motion with different accelerations
- To study kinematics of two-dimensional motion

Theory

In this experiment we will study one- and two-dimensional motion with constant acceleration by analyzing the video of a moving object using a web camera. Video clips are made from the picture frames captured by the web camera at a regular time interval. Thus, we can determine the position of the moving object at different time intervals. From the information of the positions with respect to time, we can study kinematic quantities such as **displacement**, **velocity**, and **acceleration** of the moving object, and the relationships among these quantities.

In one-dimensional motion, object moves along a straight line, for example, along the x-axis. If the object is changing its position uniformly with time it is in *constant velocity motion*. **Velocity** is defined as the time rate of change of displacement. In the case of motion with constant velocity, the position of the object (x) at time, t , can be determined as:

$$x = x_o + vt \quad (1)$$

where x_o is the initial position and v is the velocity. The quantity $(x - x_o)$ gives the displacement.

If the object is speeding up or down uniformly, it is in constant acceleration motion. Acceleration is defined as the time rate of change of velocity. In one-dimensional motion with constant acceleration, velocity (v) of the object at any time, t , is given by

$$v = v_o + at \quad (2)$$

where v_o is the initial velocity and a is the acceleration. This is a linear equation of velocity versus time. Therefore, *the graph of velocity versus time will be a straight line* whose slope gives the acceleration.

In order to find the position of the object in an accelerating motion we cannot use equation 1 because the velocity is changing. For a motion with constant acceleration, since the velocity is changing uniformly, we can replace the velocity, v in equation 1 by an average of the final velocity, v and initial velocity, v_o , and using equation 2. That gives the position (x) of the object at time, t in a constant acceleration motion as:

$$x = x_o + v_o t + \frac{1}{2}at^2 \quad (3)$$

This is a quadratic equation of position versus time. Therefore, *the graph of position versus time will be parabolic*. Note: one dimensional motion has two possible directions. So, the positive and negative values have particular meanings.

A free fall of an object due the gravity is an example of motion with constant acceleration. For an object on a frictionless inclined plane as shown in **Figure 1**, acceleration, a , of the object is given by

$$a = g \sin \theta \quad (4)$$

where, θ is the angle of inclination. By changing θ , acceleration can be changed.

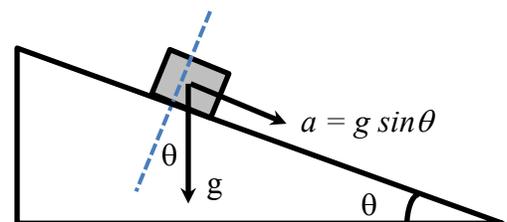


Figure 1. Acceleration on a frictionless inclined surface.

We will also study kinematics of two-dimensional motion. Two-dimensional motion can be considered as the combination of two one-dimensional motions along mutually perpendicular directions such as along the x-axis and y-axis. Projectile motion is a good example of two-dimensional motion. In the absence of air resistance, a projectile moves with constant velocity along horizontal direction (or x-axis) whereas it moves with constant acceleration of g (directed downward) along vertical direction (or y-axis). We will study two-dimensional motion similar to a projectile motion of an object on the surface of air table that will have constant acceleration of $a = g \sin\theta$ along the direction of inclination and no acceleration perpendicular direction to it.

Apparatus

Air table with blower, puck, meter stick, rectangular wooden block, webcam, and computer with Logger Pro software.

Description of Apparatus

This experiment is performed with a puck on an air table as shown in **Figure 2a**. The air table is essentially a box with many small holes on the top surface. An air blower supplies air into the box through a pipe connected to the box. The air coming out from the several holes makes the surface 'frictionless'. When the puck is placed on the air table the puck floats on those small air fountains, which reduces the friction. Thus the motion of the puck on the air table can be considered as motion on frictionless surface. A webcam is placed over the air table. The webcam records the video which is essentially the pictures of the moving puck captured at a regular time interval. We will use the picture frames to determine the positions of the puck at different times and analyze its motion.

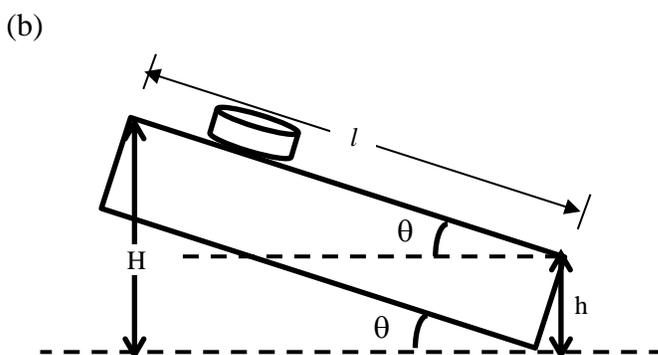
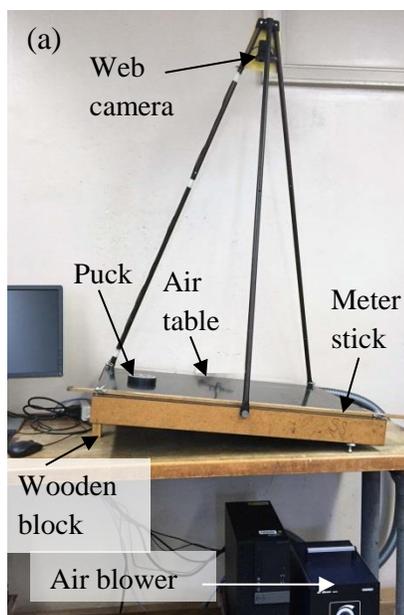


Figure 2: (a) Experimental set up, (b) Geometry to calculate angle of inclination

As shown in the geometry in the **Figure 2b**, by measuring the height of the edges of the table and the length of the table, the angle of the inclination can be calculated as

$$\sin\theta = \frac{H-h}{l} \quad (5)$$

where H and h are the heights of the upper and lower edges of the table, and l is the length of the table.

Procedure

Getting Ready: Before starting the experiment make sure the air table is leveled. Put the puck at the center of the table and turn on the blower. If the table is leveled, the puck should remain motionless or move very slowly. Otherwise, level the table by adjusting the screws at the bottom of the table. Turn off the blower.

The computer should already be turned on. Open Logger Pro program. An empty graph will show up with a blank table on the left side of the window. On the menu bar, click 'Insert' and select 'Video Capture'. Video Capture window should show the image of the whole top surface of the air table. If the air table doesn't fit in the window, ask the instructor to assist.

Part I. Puck sliding down freely

1. You are given a rectangular block that has three different heights on its sides. Put the smallest height of the rectangular wooden block under the center leg of the table edge. Measure the height of the edges and length of the table to calculate the angle of inclination.
2. Put the meter stick with two marks separating by 0.5 m apart along the edge of the table. This will be used to establish the scale for the video pictures. Put the puck near the middle of the higher end of the table and under the string at the border. The string should hold the puck.
3. Turn on the blower. Click on 'Start Capture' and when it is replaced by 'Stop Capture' gently lift the border string to release the puck. The webcam should be recording a video while the puck is sliding down. Click 'Stop Capture' when the puck hits the lower border string. Turn off the blower and close the Video Capture window.
4. You should have the video of the puck in a small frame in the Logger Pro window. Enlarge the frame size by dragging by the centers or corner of the edges. Play the video to see what you have. The icons: ▶, ■, ◀▶, ▶▶, ◀◀ on the bottom of the frame mean 'play', 'stop', 'move to the first frame', 'move forward frame by frame', and 'move backward frame by frame' respectively. Set the video at the frame where you wish to start analysis.
5. **Calibration of length:** Click on the icon ◯◯▶ 'Enable/Disable' video analysis at the bottom on the right. A column of tool icons will appear on the right side of the frame. Click on 'Set Scale' (fourth from top). Now, bring the cursor on the frame to a mark on the meter stick, hold the mouse button and drag cursor to the other mark and release the button. Scale window will show up. Type 0.5 (it is the distance between the marks) and click 'OK'. The scale is set.
6. Click on 'Add Point' (second from top). The cursor will appear as + sign. In the first picture frame, bring the cursor to a particular point such as center of edge of the puck and click the mouse. It will add coordinates (X and Y) and corresponding time in the table as well as points on the graph, and then advances to the next frame.

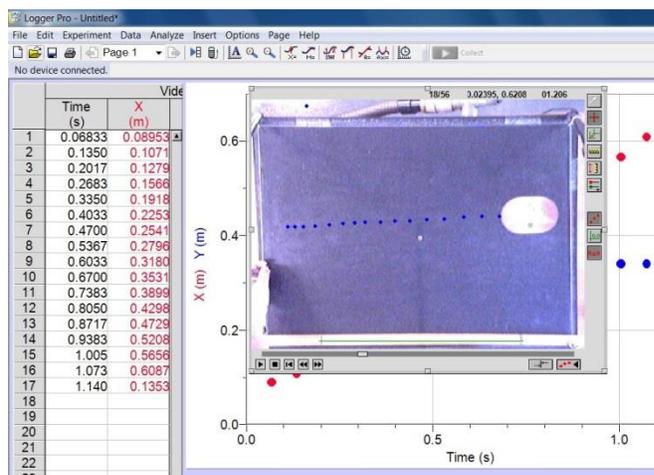


Figure 3: Screen shot while adding points

Again bring the cursor to the same point of the puck and click. Continue until you have recorded for all the frames in your analysis, i.e. before the puck hits the bottom border string. In the table you can also see calculated values of X and Y components of velocities.

What is the time interval of the frames? How many frames are captured in 1 second?

7. The program takes lower left corner as the origin. You can assign a particular position as origin. Click on 'Set Origin' (third tool from top). Move the cursor on the point where you choose as the origin and click at the center of the puck (It could be the first point). Click on the graph area under the frame and we have *the graph of position X and Y vs. Time*.

What is the nature of the position (X) and (Y) versus time graphs? Does it make sense according to equation 3?

8. **Analysis of the graph:** Since it is one-dimensional motion you need either X or Y on the vertical axis in the graph. If X coordinates are changing, you can replace the graph of Y by X velocity. Right click on Y in the vertical axis and choose X-Velocity. You will get the graph of X and X-velocity versus time graphs. (You can also change it from menu bar of the window. Click 'Experiment' choose 'Graph Options' and then select 'Axes Options' and choose X and X-Velocity in Y-Axis Columns, and click 'Done'.) Change the scale of the graph if needed. You may use 'Autoscale' or go back to 'Graph Options' → Axes Options, and manually insert the range of the scale to display all the points and click 'Done'.

Now, on the graph, highlight the portion of the graph to analyze using the mouse (You may exclude the points near the border which may not fit well). In the menu bar, click 'Analyze' and select 'Linear' for velocity versus time graph. For position versus time graph, select 'Curve Fit' and choose quadratic function. Click 'Try fit'. Check if the fitted line coincides with experimental points, and click 'OK'. Next to the graph a small window will appear with equation and values for the coefficients. Compare the fitting equations with Equation 2 or 3 to know what the values of the coefficients are.

What do you get from the coefficient, A, by fitting velocity versus time graph?

What do you get from the coefficient, A, by fitting position versus time graph?

Write down the quantities from the fitting in the data sheet.

9. Print or save the Graph for your report. (You may include your name and comments like height of the wooden block in the Printing Options). You need only one graph from one of the trials.
10. Change the height of the wooden block under the leg to *intermediate height to increase the angle of inclination*. Measure the heights of the edges and write down the heights on the data sheet.
11. Repeat the steps 2 to 8 for this inclination.
12. Repeat the trial by changing the height of the wooden block under the leg to *maximum height*.

Part II. Puck moving up and down

In this part of the experiment, you are going to study the motion of the puck while it is moving up from the bottom the table and coming back to the bottom.

13. Change the wooden block under the leg to intermediate height of the block. Follow the steps in Part I. In this experiment, you are releasing the puck from the higher edge of the table as in Part I, but the video frames you are going to use for the analysis are *after it bounced from the lower*

string and before it hits the lower string again. So, click ‘Stop Capture’ after the puck hits the lower border string second time.

Part III. Two-dimensional motion of the puck

In this part of the experiment, you are going to launch the puck as a projectile on the air table. You need to analysis the kinematic quantities in both directions.

14. Change the height of the wooden block under the leg to *the largest height*. Practice launching the puck with different velocities until you can launch it reliably with an initial velocity such that it moves in a trajectory covering a significant fraction of the air table. It should reach about $\frac{3}{4}$ of the way up the table and travel at least half the width of the table. Once you are comfortable in launching the projectile, capture the motion of the puck and analyze it as in Part I. Initial frame to analyze should be when the puck is off your hand.

In this part of the experiment, you need to analyze graphs of position, X and X-velocity versus time, and position Y and Y-velocity. Print or save both graphs and include in your report.

How do position X and X-velocity change with respect to time?

How do position Y and Y-velocity change with respect to time? Why are the graphs different?

Computations

In table 1 for part I, calculate sine of the angle of inclination for all trials.

Determine the acceleration of the puck on the inclined air table from the fitting values of from the graphs of position and velocity.

Calculate the value of acceleration due to gravity using acceleration and angle of inclination using Equation (4). Compare the average experimental value of the acceleration due to gravity with $g = 9.80 \text{ m/s}^2$. Calculate the percent error.

Discuss the graph of position versus time and velocity versus time for all three parts of the experiments. Explain the physical significance of the values of the coefficients obtained by graph fittings.

Questions

1. In part I, instead of releasing the puck in the beginning of the motion, if it was pushed downward, how do the position and velocity curve vary? Will the acceleration increase or decrease?
2. At the highest position of the puck in Part II, velocity is zero. Is the acceleration zero? Explain your answer.
3. If the air table is placed horizontal and leveled, and the puck is pushed gently, what will be the nature of the graphs of position, velocity and acceleration versus time?
4. Which of the angle inclinations of the air table (large or small) has better agreement to the acceleration due to gravity, g , in your experiment? Explain the reasons.

Data Sheet

Date experiment performed:

Name of the group members:

Length of the air table = m

Part I: Puck sliding down freely

Table 1

Trail #	Wooden Block height	Height of the edges of table		$\sin \theta = \frac{H-h}{l}$	From position graph, a (m/s ²)	From velocity graph, a (m/s ²)	Average, a (m/s ²)	g (m/s ²)
		H	h					
1	Smallest							
2	Intermediate							
3	Maximum							

Average value of acceleration of gravity =

% error =

Part II. Puck moving up and down

Table 2 (wooden block with intermediate height)

Trial	From position graph, a (m/s ²)	From velocity graph, a (m/s ²)	Average, a (m/s ²)	g (m/s ²)

Acceleration of gravity, g =

% error =

Part III. Two-dimensional motion of the puck

Table 3 (wooden block with maximum height)

Axis	From position graph, a (m/s ²)	From velocity graph, a (m/s ²)	Average, a (m/s ²)
X	$a_x =$	$a_x =$	$a_x =$
Y	$a_y =$	$a_y =$	$a_y =$

Acceleration of gravity, g =

% error =