RADIOACTIVITY

Purpose

To study the concept of half-life in a simulated radioactive specimen and to observe statistical variation in radioactive decay.

Theory

One way of looking at a nucleus is to think of it as a box, in which there are many particles, each with some energy. These particles continually exchange energy and, in some cases, an individual particle may acquire sufficient energy to penetrate the walls of the box. In the case of stable nuclei, the particles cannot get out of the box and the nuclei "live" forever. In the case of unstable nuclei, a particle does eventually get out of the box. Some unstable nuclei have high, thick walls - these are the long-lived nuclei. Others have low, thin walls - these are the short-lived nuclei, those that have a high probability of emitting particles and changing into different, more stable nuclei.

Radioactive decay appears to be a random process. For all radioactive isotopes one finds that the "decay" curve (i.e., the plot of the number of disintegrations per second as a function of time) appears as shown in the graph below (Fig.1). When there are very many radioactive nuclei in a sample, then the number of disintegrations per second can be described extremely well by a probability curve. Even though each decay is a random event, the totality of events is described by a well-defined equation. In the case of nuclear decay, the equation describing the number of nuclei remaining at time t is given by an exponential function and the graph showing how the number of radioactive nuclei decreases with time is called an EXPONENTIAL DECAY CURVE. (See Fig. 1.) Note that it approaches zero in the limit as time increases. Frequently, it is more convenient to express the exponential decay in terms of the "half-life", $T_{1/2}$, of the nucleus, the time required for half of the nuclei to disintegrate.

Moreover, after an interval of **two** half-lives has elapsed, half of the nuclei still present at the end of the first half-life will have decayed, leaving one-fourth of the original amount. Thus with the passage of each successive half-life of time, the number of remaining radioactive nuclei has decreased by a factor of two. This is known as the RADIOACTIVE DECAY LAW. It should be recognized that it is a statistical law; it becomes less and less accurate as the number of radioactive nuclei gets smaller. The half-life of a given radioactive isotope is an absolutely fixed quantity; it does not change under any conditions. Half-lives run the range from billions of years (for a species that decays at a very slow rate) to fractions of a second (for a highly unstable nucleus). Note that the "clock" can be started at any instant. Hence in a time $T_{1/2}$, half the starting number of nuclei will have decayed regardless of when the clock was started.

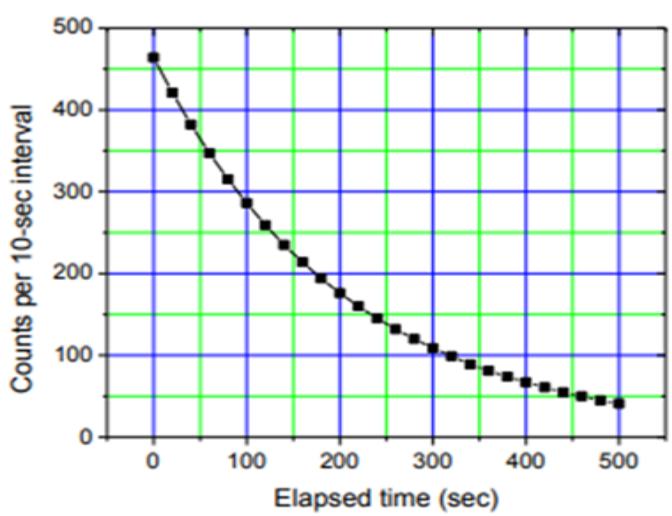

Fig. 1. Typical plot of radioactive decay.

Simulation

This lab uses an online simulation from Andrew Duffy (Boston University), available at this URL: http://physics.bu.edu/~duffy/HTML5/halflife.html
The simulation should work in any web browser as it is a HTML5 simulation.

Description of Simulation

This simple simulation examines the decay of 400 radioactive nuclei. As the simulation author, Andrew Duffy writes:

> *This is a simulation of the radioactive decay of 400 radioactive nuclei. You can choose from three different half-lives. Note that the nuclei turn blue when they have decayed, and the smooth purple line on the graph shows the ideal case. An actual case can deviate a bit from the ideal case because of statistical fluctuations (which make more of a difference when the number of nuclei is small, as it is here).*

We will examine two aspects of the simulation:

- *The half-life of a radioactive nucleus*. Three different half-lives are available in the simulator (short, medium, long). You will record and plot the number of undecayed nuclei as a function of time for **two of these cases** (long and medium half-life). The type of radioactive decay (alpha, beta or gamma radiation) is not specified.
- *The statistical variation of time-dependent decay*. This will be seen from re-running the simulation and recording the variation in decay statistics between runs.

Procedure

a. Finding the half-life of a nucleus by recording the number of undecayed nuclei for two cases

Open the simulation at the above URL. Note the axes of the plot: the vertical axis is the number of *undecayed* nuclei (also shown as 400 red dots below the plot when the simulation first begins). The horizontal axis is time in seconds (although the time unit could in principle be hours, days or even years!)

1. Select the nucleus with the largest half-life by clicking *Larger* in the "Half-Life" options (2^{nd} row of buttons below the plot).
2. Note that, at time t=0 seconds, the number of undecayed nuclei, N=400.
3. Press *Step* >> and observe that the time at the top right of the screen increments by 1 second. The value of N falls slightly.
4. Press *Step* >> repeatedly, noting both the values of time (t) and undecayed nuclei (N) every 4 seconds in *Table I*. Stop recording data once you reach 100 seconds. Press *Reset* to clear the plot.
5. Select the nucleus with the medium value of half-life by clicking *Medium* in the "Half-Life" options.
6. Repeat steps 2-4, again recording values of t and N every 4 seconds for 100 seconds, but now in *Table II*. Notice that the decay in this case is faster than for the largest half-life nucleus. Once complete, press *Reset* to clear the plot.

Computation for part (a) [to be completed before part (b)]

1. *Plot* a graph with **N**, the number of undecayed nuclei, on the y-axis and **t**, the time on the x-axis.
2. The data points will show some scatter on your graph. Nevertheless, you should be able to draw a *smooth* curve, *resembling* the exponential decay curve shown in Fig. 1, which represents the "best fit" to the data.
3. The half-life of the simulated nucleus can now be determined from the graph, using the definition of the half-life as the time it takes for the radioactive decay rate to fall by a half. The idea is to *select* a pair of numbers of undecayed nuclei, $\mathbf{N_1}$ and $\mathbf{N_2}$, such that $\mathbf{N_2}$ is *half* of $\mathbf{N_1}$. Using the curve you have drawn for the longest half-life nucleus (data from Table I), *read off* the corresponding times $\mathbf{t_1}$ and $\mathbf{t_2}$, estimating to the nearest half-second. **Enter these values for the longest half-life nucleus in *Table III***. The difference $\mathbf{\Delta t = t_2 - t_1}$ represents the time taken for the nuclear activity to decrease by a half. It should be *calculated* and *recorded* in Table III.
4. *Repeat* this process for 4 additional pairs of points, again for the same data set (longest half-life nucleus). Then *compute* the mean value of $\mathbf{\Delta t}$ and *record* it below Table III. *Calculate* the deviations of $\mathbf{\Delta t}$ from the mean and *record* them in the last column of Table III. *Calculate* the AD (Average Deviation) and *record* it below Table III.
5. Using the method in steps 3 and 4, make the same calculation of half-life for the medium half-life nucleus (data from Table II) using pairs of numbers of undecayed nuclei. Enter your chosen values of $\mathbf{N_1}$ and $\mathbf{N_2}$ (where $\mathbf{N_2}$ is *half* of $\mathbf{N_1}$) and the corresponding values of time ($\mathbf{t_1}$ and $\mathbf{t_2}$) in *Table IV.* Using a total of 5 pairs of points, *compute* the mean value of $\mathbf{\Delta t}$ and *record* it below Table IV. *Calculate* the deviations of $\mathbf{\Delta t}$ from the mean and *record* them in the last column of Table IV. *Calculate* the AD (Average Deviation) and *record* it below Table IV.

b. Observing statistical variation in radioactive decay.

Having taken the decay data in part (a) above, and having made the computations in part (b), you should now have a value of the half-life of each of the two nuclei that you studied: the "longer" and "medium" half-life nuclei.

To conclude the experimentation, we will now re-run the simulation for *one of the two nuclei that you studied in part* (a) in order to observe the variation in decay statistics from run to run.

1. Firstly, make sure that the simulation is starting from a "reset" state with N=400 by pressing "Reset". Click "Larger" or "Medium" half-life (your choice) but make sure that you have already calculate/estimated the half-life of whichever nucleus you choose.
2. Next, advance the simulation slowly by pressing *Step* >> repeatedly until you reach the time that corresponds closest to the half-life calculated in part (a). Since your calculated half-life will likely not be an integer value of time, you should stop the simulation at the nearest integer value of time in seconds. Note the value of **N** at this value of time in *Table V*.
3. Reset the simulation so that N=400.
4. Repeat steps 1-3 for a total of 10 runs, stopping the simulation on each occasion at the time that is closest to the calculated value of half-life, noting the value of **N** at that time.

Computation for part (b)

If each run was perfectly reproducible, then exactly 200 nuclei would remain undecayed after 1 half-life. From your data in Table V, does the value of N ever exceed 200 when 1 half-life has elapsed? If so, how many times does this occur in the 10 runs that you record? What does your observation tell you about the nature of radioactive decay?

Questions

1. Suppose a radioactive nucleus has a half-life of 2 min. and suppose the counting rate at time $t = 0$ is 3000 counts/10 sec.
    a. What is the counting rate after 2 min?
    b. After 6 min?
    c. After 10 min?
    d. After 20 min?
2. Isotope X has a half-life of 100 days. A sample is known to have contained about 1,500,000 atoms of isotope X when it was put together, but is now observed to have only about 100,000 atoms of isotope X. **Estimate** how long ago the sample was assembled.

Data Sheet

Date experiment performed:

Name of group members:

Note: Your graphs should be part of your data sheet.

Table I: Largest half-life nucleus

Time, t (s)	N	Time, t (s)	N	Time, t (s)	N
0		36		72	
4		40		76	
8		44		80	
12		48		84	
16		52		88	
20		56		92	
24		60		96	
28		64		100	
32		68			

Table II: Medium half-life nucleus

Time, t (s)	N	Time, t (s)	N	Time, t (s)	N
0		36		72	
4		40		76	
8		44		80	
12		48		84	
16		52		88	
20		56		92	
24		60		96	
28		64		100	
32		68			

Table III: CALCULATION OF HALF-LIFE from Table I

N_1	t_1	N_2 (= $N_1/2$)	t_2	$\Delta t = t_2 - t_1$	Deviation

HALF-LIFE (time): MEAN Δt ______________ AD ________________

CALCULATED HALF-LIFE = ($\Delta t \pm$ AD) = ___________±_______________sec

Table IV. CALCULATION OF HALF-LIFE from Table II

N_1	t_1	N_2 (= $N_1/2$)	t_2	$\Delta t = t_2 - t_1$	Deviation

HALF-LIFE (time): MEAN Δt ______________ AD ________________

CALCULATED HALF-LIFE = ($\Delta t \pm$ AD) = ___________±_______________sec

Table V: Variation in decay statistics between runs

Choice of nucleus: Medium half-life / Larger half-life [select one]

Half-life of this nucleus (calculated from Table III or Table IV): $t_{1/2}$ = ___________±__________sec

Run #	N after 1 half-life	Run #	N after 1 half-life	Run #	N after 1 half-life
1		5		9	
2		6		10	
3		7			
4		8			