Constant speed & graphs

Purpose

1. To learn how speed is related to distance and time and explore a method using photo gates to measure constant speed.
2. To plot a graph of distance versus time, find the slope, and interpret it as a speed.

Theory

In one-dimensional motion the object moves along a straight line, for example along the x direction. If the object is changing its position *uniformly* with time it is in **constant-velocity motion**. Velocity is defined as the time rate of change of displacement. In the case of motion with constant velocity, the position of the object (x) at time, t, can be determined as:

$$x = x_o + vt \qquad (1)$$

where x_o is the *initial* position and v is the velocity. The quantity $(x - x_o)$ gives the *displacement*. So, from Eq. (1), $v.t = (x - x_o)$ and therefore

$$v = \frac{(x - x_o)}{t} \qquad (2)$$

or, in words, speed = distance travelled /time elapsed. In today's lab the distance that we need to use is the distance traversed *between two positions*; i.e., the *change* in the distance. So we should really say

speed = *change* in distance /elapsed time (3)

(That is why we write $\boldsymbol{x_o}$ in Eq. (1) and $x - \boldsymbol{x_o}$ in Eq. (2).)

If we **let the *displacement,* $\boldsymbol{x - x_o}$, be represented by *d*** (standing for the **d**isplacement, or change in **d**istance) then we may write

$$v = d\, t \qquad (4)$$

In today's lab we will measure different values of *d* and *t* and present our data in a tabular form. We will use these data to plot a graph of *d* versus *t*. Since $v = d/t$ or, equivalently, $d = vt$, the speed v is the **slope** of the d vs. t graph which, theoretically, would be a *straight line*.

Running the experiment (**The data sheet for the experiment is on page 3 & appendix on graphs is on pages 4 & 5**)

Measuring constant speed

1) Open the simulator https://www.walter-fendt.de/html5/phen/acceleration_en.htm

Read the explanatory text written at the top of the simulator window.

2) Set the acceleration to 0 m/s^2. Set the initial velocity to 2.00 m/s. Keep the initial position 0.00 m.

3) When you run the simulator all timers will count time starting from zero seconds. The car will move and as it passes through the green photo gate, this green photo gate will stop indicating the time at which the car reached it. Let us call this time t_1. The red photo gate is still running until the car reaches it, then it will stop counting. The reading of the red photo gate is the time for the car to reach that gate starting from 0 meters. Let us call this time t_2

4) Click and drag the green photo gate to set it at a position of 15 m. Let us call the position of this green gate x_o.

5) Click and drag the red photo gate to set it at a position of 30 m. Let us call the position of the red photo gate x. See figure 1.

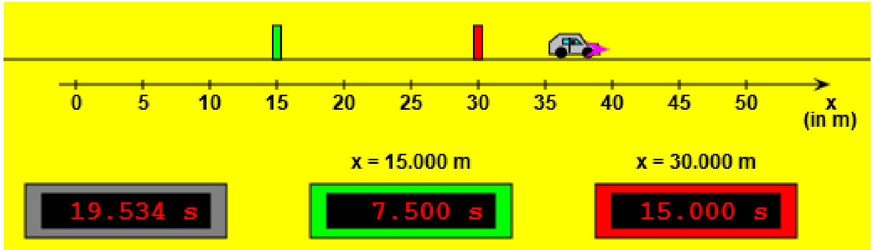

Figure 1: Positions of green and red photo gates for trial 1. Image courtesy of Walter Fendt: https://www.walter-fendt.de/html5/phen/acceleration_en.htm

6) Click Start. Observe the photo gates and the graphs of position and velocity (here we can regard the value of the velocity as the speed). You can 'Pause' the simulation after the car crosses the red photo gate. Record the time indicated by the green and the red photo gates in table 1.

7) Change the position of the red photo gate to 35 m, then 40 m, 45 m, 50 m. Each time repeat step 6. Now you should have a total of 5 entries for times of red photo gate. Record them in the table in the data sheet.

8) To find the time for the car to cross the distance between the green and red photo gates, simply subtract the time recorded by the green photo gate, t_1 from the time recorded by the red photo gate, t_2:

$$\Delta t = t_2 - t_1$$

9) To find the distance traveled by the car between the green and red photo gates simply subtract the position of the green gate, x_o from the position of the red gate, x:

$$d = x - x_o$$

10) Plot a graph of distance travelled d (on the y axis) versus Δt (on the x axis). Compute the slope. What does the slope represent?

11) For each row in the table, calculate and record the speed (show your work):

$$v = \frac{d}{\Delta t}$$

12) Notice that our values here are exact, since the simulator uses the theoretical equations in its programming code. If we were to perform the experiment in the lab, we would find that there are some fluctuations in the measured values and then we would take an average as our best estimate and there would be an uncertainty representing the spread of the measured values, as was explained in the last experiment, experiment 1.

If the experiment was performed in the lab, there will also be a small difference between the value of the speed calculated from the slope of the graph and that calculated from average of $\frac{d}{\Delta t}$ values. This is because the graph (best fit line) would then minimize error.

Questions

1. A cyclist bikes 215 km in 5.1 h (hours) at constant speed. What is her speed?

2. A gun shoots a bullet with a speed of 612 m/s. How long does it take the bullet to hit target that is 2965 m away?

Data Sheet

Name: **Group:** **Date experiment performed:**

Measuring constant speed

Position of green gate x_o (m)	Position of red gate x (m)	$d = x - x_o$ (m)	Reading of green gate t_1 (s)	Reading of red gate t_2 (s)	$\Delta t = t_2 - t_1$ (s)	$v = \frac{d}{\Delta t}$ (m/s)
15	30					
15	35					
15	40					
15	45					
15	50					

Calculations of $v's$ (show your work here):

Step 10) Slope of graph of d versus Δt:

Answers to questions:

1.

2.

Remember to always write the correct units and remember to include your graph of d versus Δt in your lab report and to show the slope calculation on the graph.

APPENDIX ON GRAPHS

Quantities depend on other quantities: Your weight depends on your food intake, the Dow Jones average depends on interest rates, your grades depend on how hard you study, and the distance a ball rolls depends on how long it has been rolling.

Of course, for all the above cases, life can be (and usually is) more complicated; other factors may enter in to influence your weight, the stock market, your GPA, or that rolling ball. But in the science game, we try to begin to study quantities by looking first at the *major factors* on which they depend.

Once we see that a quantity (such as distance) seems to depend on another quantity (such as time) in a significant and reproducible way, we can start to get systematic. The first step is to measure distances for a number of times and to make a table of the results.

Imagine that we now have a table of five pairs of numbers and that this table gives us the totality of information we have on this particular subject. Are we finished? Have we gotten as much as we can out of our data? The answer is no. We have the data. We have even organized it systematically. But we have not presented it to ourselves (or the scientific community) in the form that speaks most strongly to the imagination, i.e., that allows us to see the possibility of a *pattern* that might exist in the way distance depends on time. For that, we need to draw a **graph**.

Graphs are mysterious things. When these five pairs of numbers are turned into five dots on a graph, something magic happens. We start asking questions: Do these points lie in a straight line? Do they form a curve? If I measure a new pair of points in a certain region of the graph, where would it lie? Do the points suggest a simple mathematical relationship between distance and time?

So a graph is a stimulus for creating generalizations from a limited amount of specific information. It is *not* a tool for constructing an absolute proof of anything, because in forming our generalizations (or "laws") we are making best guesses from looking at a finite number of data points, each of which, because it is measured, has a built-in uncertainty. The process of making such guesses is called *induction*, and it always carries some risks. Nevertheless, this is the tool with which science carves out new laws, and it has served us well.

PLOTTING THE DATA: The first step in plotting a graph is to **choose scales** for each axis, i.e., you must select appropriate values for the smallest divisions or "boxes" on the graph paper. Start by drawing the vertical and horizontal axes on the paper, so that the origin is (usually) in the lower left hand section of the graph paper. (See Figure 2) Choose the divisions so as to make the graph *as large as possible* (Avoid the "postage stamp" graph!). A good rule of thumb for the graphs in these labs is to choose the divisions so that the *largest point* (highest values of distance and time) on the graph falls somewhere in the upper right-hand quadrant, as shown in Figure 2 (You may have to try more than once to get the best presentation, whether landscape or portrait, so use pencil and eraser!).

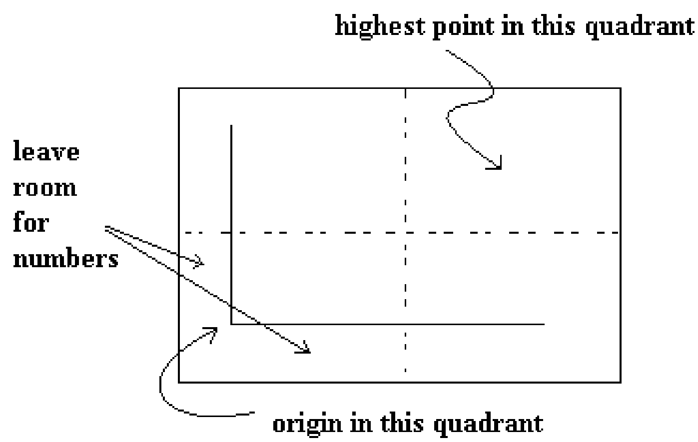

Figure 2: Setting up a graph

Now label the axes with the quantities being plotted (e.g., distance and time), including their units. Mark the position of each data point as carefully as you can with a small circle, a dot, or a dot enclosed by a small circle. Note the uncertainty of each point, as obtained when you measured it. If the uncertainty is large enough to plot (larger than the size of the dot or circle), indicate its size on the graph with a small vertical or horizontal line through the dot or with an error bar. Note that uncertainty can exist along either axis (e.g., significant uncertainty in both time and distance), so you might

end up with a small cross through the point or with two perpendicular error bars. Note that in our simulation lab values were exact.
The next job is to guess at the pattern which the dots seem to form, which may lead us to hypothesize a simple mathematical relationship between the quantities involved. This is called “fitting” the data to a curve, where the curve is the graphical expression of the assumed relationship. Note that, because of uncertainty in the data, the points may not fall *exactly* on a smooth curve or line, but may tend to scatter slightly. Nevertheless, experience has taught us to look in general for a ***smooth*** curve that comes *close* to the points, rather than to play connect-the-dots.

PROPORTIONALITY. In these labs, we will *mostly* be trying to ***‘fit’*** our data with one simple type of curve—that of a *straight line through the origin* of the graph. Such a straight line is the graphical expression of a simple mathematical relationship between variables—that of *proportionality*.
It is the assumption of proportionality that *leads us to include the origin in our straight line fit*: After all, if we think the distance a car moves may be proportional to the time it has been moving, it is completely reasonable to assume that if no time has passed (***t*** = 0), then the ball has not moved any distance (***d*** = 0), *even if we have not actually been able to measure this point*.
So how do we “fit” a straight line to our data points? There are two approaches. Which one we use should depend on how well the points line up on the graph.

In our case all the points should come out to be perfectly lying on a straight line passing through the origin point, (0,0). This is because, as mentioned in the experiment, our measurements are based on an exact simulation. The graph should look like figure 3.

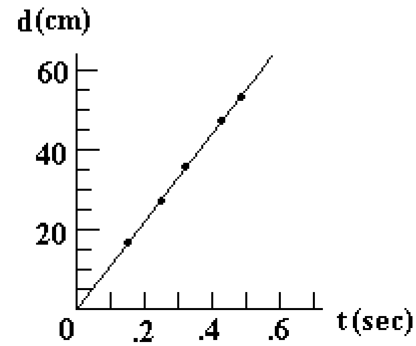

Figure 3: Example of graph with all points lining up along a straight line

The slope of the line is a number of considerable interest to us, because it is a ***constant*** for a straight line, and it has been entirely determined by our measurements. In fact, the mathematical translation of the statement “*d* is proportional to *t*” is just the equation

$$d = Kt$$

where K is some constant and is called the *constant of proportionality* in the equation.
As you might have guessed, K is just *the* ***slope*** *of our straight line*.

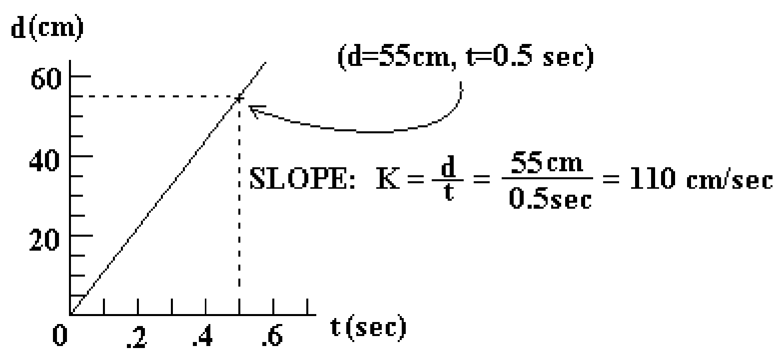

Figure 4: Finding the slope of a straight line

So once we measure it, we have completed the job of translating our “best fit” pattern into a mathematical expression.
The slope can be measured from the graph as follows: select any point on the line you have drawn (but ***NOT*** the data points), *preferably in the* ***upper half*** *of the graph for accuracy*, and draw vertical and horizontal dashed lines to the axes, in order to find the coordinates of your selected point, as shown in the example in Figure 4.

Note we have not drawn the data points, to emphasize that we are *using the* ***line*** to measure the slope. Locate the coordinates (value of ***d*** and ***t***) of the point (marked on the graph with a **+ mark**)

In an actual lab we encounter another case where not all the points lie on the straight line. In that case we draw the best fit. If the points seem to fit a straight line then draw the straight line that goes through or close to all the points (or error bars if there are any). If it doesn’t, make an “eyeball” fit of the data. A rule of thumb is that you have about as many points above your line as below your line. Measure the **slope** of that line. This gives you the “best” value.