Speed and constant acceleration

Purpose

1. To study kinematic equations, calculate the constant acceleration, a and compare to simulator value.
2. To graph kinematic equations, and relate slope to the constant acceleration value.

Theory

We've already seen one kind of speed in the first two labs. We call it the *average speed*. It's the kind of number you get when you take a car trip of say 200 miles, and divide that distance by the time it takes for the trip, say 4 hours. Your average speed is then 50 miles per hour for the whole trip. But what about the kind of speed you see on your speedometer as you pass that state cop sitting behind a tree? That isn't always 50 mph….sometimes it's faster, sometimes slower. That kind of speed, the kind you have as you pass a point (or a state cop) is called the *instantaneous speed*. It is obviously different from the average speed, because it can change as you go around a curve or slow down behind a truck or speed up on a straight road.

The instantaneous speed, v, at a point is something that is impossible to measure exactly. But we can come close to it by measuring the average speed *over a very short distance surrounding the point*. That's what a speedometer does in your car. So we may estimate the instantaneous speed by

$$v = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t} \sim \frac{very\ short\ distance}{very\ short\ time} \qquad (1)$$

The instantaneous speed is the positive value of the instantaneous velocity. The difference is that the instantaneous velocity has a direction, while the speed has no direction. Also displacement d which is the change in position, Δx has a direction, while distance has no direction and is always positive. For motion in a fixed direction the distance is the positive value of the displacement. For a motion on a straight line, the instantaneous velocity as well as the displacement could be in one of two opposite directions. If one direction is chosen to be a positive direction, then the opposite direction is taken to be a negative direction. In our experiment we will study an object moving in a fixed direction.

If an object **starts from rest** and moves with constant acceleration, a, the square of its instantaneous speed, v, is proportional to its displacement (change in position), d, with the proportionality constant being twice the acceleration:

$$v^2 = 2\,a\,d \qquad (2)$$

It turns out that there is a more fundamental relationship between the *speed* of the object moving with constant acceleration, a and the *total **time*** that has passed since it was released. In motion with constant acceleration, when **starting from rest** the speed is proportional to the time with the proportionality constant being the acceleration:

$$v = a\,t \qquad (3)$$

In addition, for an object starting from rest, the displacement which is the change in position Δx is proportional to the square of the time. The constant of proportionality is one-half the acceleration:

$$\Delta x = \tfrac{1}{2} a\, t^2 \quad (4)$$

If the object does not start from rest but **has an initial non-zero velocity** v_o, then eqn. 4 becomes:

$$\Delta x = v_o t + \frac{1}{2} a\, t^2 \quad (5)$$

Equations 2 to 5 describe motion with constant acceleration and belong to equations known in physics as kinematic equations.

Description of the experiment:

Consider a car starting from rest and having a constant acceleration, a. Since the acceleration is constant we can use the kinematic equations. Along the path of the car we have two photo gates placed at known positions, x_1 for 1st photo gate (green) and x_2 for 2nd photo gate (red). The photo gates record the time for the car to reach each of them starting from time $t = 0\ s$ at the beginning of the motion. Let us call the time for the car to reach the first photo gate, t_1 and the time for the car to reach the second photo gate, t_2. See figure 1.

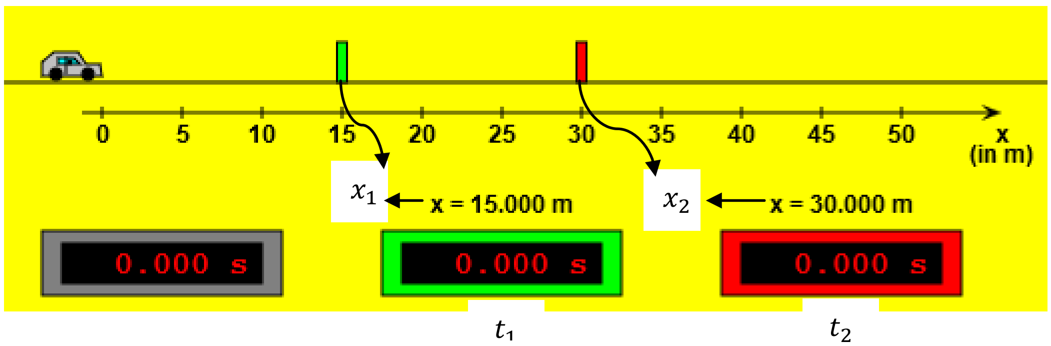

Figure 1: Car moving with constant acceleration and crossing two photo gates. Image from simulation courtesy of Walter Fendt: https://www.walterfendt.de/html5/phen/acceleration_en.htm

We want to calculate the acceleration. Since the acceleration is constant here for all times, let us focus on the motion for the interval starting at the first photo gate (green) and up to the second photo gate (red), since the acceleration is constant also during that interval. We do so because we have some useful givens in this interval. We have the time of travel between the photo gates $\Delta t = t_2 - t_1$. We will call Δt as just t. We also have the displacement between the two photo gates $\Delta x = x_2 - x_1$. If we try equation 5:

$$\Delta x = v_o t + \frac{1}{2}\, a\, t^2$$

we find that we have two unknowns, a and v_o. Notice that since we are focusing on the motion between the two photo gates, then the value of $v_o$is the value of the velocity of the car at the first, green photo gate. To find v_o let us consider another motion (let's call it **motion A**): the motion from the position $x = 0\ m$ and time $t = 0\ s$ and up to the position of the first photo gate (green). Notice that the final velocity of this motion is the initial velocity v_o for the motion between the photo gates. For motion A, from $x = 0\ m$ to the green photo gate, since the motion starts from rest let us apply eqn. 3:

$$v = a\, t$$

Let us call this t as t_1, the time to reach the green photo gate. Therefore, for the motion between the photo gates:

$$\Delta x = a\, t_1 t + \frac{1}{2}\, a\, t^2 \quad (6)$$

Substituting the value of Δx, $t_1$and t we get the value of the constant acceleration a.

Running the experiment

(The data sheet for the experiment is on page 4)

Show all your calculations

1) Consider again figure 1. Open the simulator https://www.walter-fendt.de/html5/phen/acceleration_en.htm

2) Set the acceleration, a to $2\ m/s^2$. Click and drag the green photo gate to set its position at $x_1 = 15\ m$. Click and drag the red photo gate to set its position at $x_2 = 30\ m$. Keep the initial position of the car at $0\ m$ and the initial velocity at $0\ m/s$.

3) For the motion of the car between the gates, from the green to the red photo gate, find $\Delta x = x_2 - x_1$.

4) Click start to run the simulator. Observe the timers of the photo gates and observe the graphs of position, velocity and acceleration. Pause the simulator at any time after the car crosses the red photo gate. Record t_1 and t_2. For this motion (between the gates) calculate $t = t_2 - t_1$.

5) For motion A, mentioned in the description of the experiment in the previous page (from $t = 0\ s$ and up to the green photo gate), we can use eqn. 3: $v = at_1$, to find the final v of motion A which, as mentioned in the description of the experiment, is the $v_o$that we need for the motion between the gates. Substitute this v and Δx (found in step 3) and t (found in step 4) in eqn. 6 to calculate the acceleration a. Compare to the given value of $a = 2\ m/s^2$.

6) Click reset in the simulator. Set the acceleration, a back to $2\ m/s^2$. Knowing the acceleration $a = 2\ m/s^2$ as a given, in the simulator click and drag to set the red photo gate at an arbitrary position x_2. We will assume that we do not know x_2. Run the simulator. Find t as in step 4, notice that $v_o = at_1$ and x_1 for this motion did not change. Use t, v_0, $x_1$and a in equation 6 to calculate x_2. Compare with the value displayed by the simulator for x_2, the position of the red photo gate. Show all your calculations.

7) Change the position of the second, red photo gate x_2 to $50\ m$ to move it out of the way. We will focus now on **motion A** (from initial $x = 0\ m$ and initial $t = 0\ s$ and up to the green photo gate. Δt for this motion is t_1 since the initial $t = 0\ s$). Change the position of the first, green photo gate $x_1$to 20, 25, 30, 35, 40 m. Each time run the simulator and record t_1 (the time to reach the green photo gate) in the table in the data sheet. Using equation 2 with $d = \Delta x = x_1 - 0$, and the value of the acceleration, $a = 2\ m/s^2$, calculate the velocity of the car, v as it reaches the green photo gate.

8) Record the values of v in the table in the data sheet. Graph v versus t (here t is t_1 the time need for the car to reach the green photo gate starting from $t = 0\ s$ and an initial position $= 0\ m$). Calculate the slope of the graph. Compare to the value of the acceleration, a.

9) Record the values of x_1 in the table in the data sheet. Compute ${t_1}^2$ and record in the table. Notice that x_1 represents $d = \Delta x$ for this motion (that we labeled as motion A in the description of the experiment) since the initial x for this motion is $0\ m$. Also $t_1$represents t since the initial $t = 0\ s$. Graph $d\ (= \Delta x)$ versus t^2 for this motion. Compare the slope to the value of the acceleration, a.

Questions

1. A rocket accelerates from rest at a rate of 50 m/s^2.
(a) What is its speed after it accelerates for 30 s?
(b) How long does it take to reach a speed of 6,500 m/s^2?

2. A skydiver jumps out of a helicopter and falls freely for 2 s before opening the parachute.
(a) What is the skydiver's, downward velocity when the parachute opens?
(b) How far below the helicopter is the skydiver when the parachute opens?

Data sheet

Name: **Group:** **Date experiment performed:**

Step 3: $\Delta x =$ Step 4: $t =$

Step 5: Calculation of v_o: Calculation of a:

Step 6: t= $v_0 =$ $x_1 =$ and a=

Calculation of x_2:

Steps 7, 8 and 9 **(Motion A)**

$x_1 = \Delta x = d\ (m)$	$t_1 = t$ (s)	v^2	$v\ (m/s)$	$t^2 (s^2)$
0	0	0	0	0
20				
25				
30				
35				
40				

Step 8, calculation of slope of v versus t graph: Compare to the acceleration, a:

Step 9, calculation of slope of Δx vesus t^2graph: Compare to the acceleration, a:

Answers to questions:

1a)

1b)

2a)

2b)