Equipotential Surface Plotting

Purpose

a. To map the *equipotential surfaces* by two charged conductors of various shapes.

b. To draw *electric field lines* produced by the two charged conductors of various shapes from equipotential surfaces. For simplicity, we will work with two-dimensional systems.

Theory

There are a few fundamental concepts from electrostatics that are involved in this lab:

Electric field lines: A convenient way of visualizing the electric field produced by a given charge distribution is by drawing lines called electric field lines. An electric field line starts at a positive charge and ends at a negative charge. The strength of the field is proportional to the density of the field lines. The tangent direction of the electric field line at any point gives the direction of the corresponding electric field, as shown in Fig. 1.

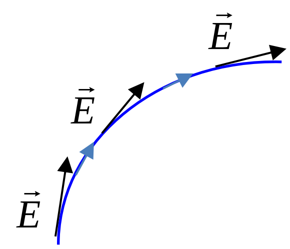
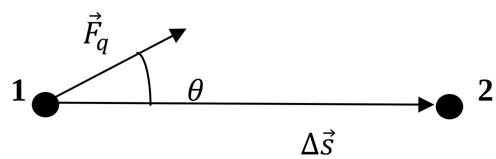


Fig. 1. Electric field line and Electric field

Electric potential (or simply, potential): The electrostatic force is a conservative force and hence, one can define a potential energy U associated to this force. For a given point charge q in the presence of an electric field $\vec{E}$, the *electrostatic potential energy* difference between two points in space "1" and "2" infinitesimally close to each other is defined by

$$\Delta U = U_2 - U_1 = -|\vec{F}_q||\Delta\vec{s}|\cos\theta \quad (1)$$

where $\vec{F}_q = q\vec{E}$, $\Delta\vec{s}$ is the displacement vector connecting points "1" and "2", and θ is the angle between $\vec{F}_q$ and $\Delta\vec{s}$:



The *electrostatic potential* difference between points "1" and "2" is defined by $\Delta V = \Delta U/q$. In other words,

$$\Delta V = V_2 - V_1 == -|\vec{E}||\Delta\vec{s}|\cos\theta \quad (2)$$

It follows that for any point charge q, the electrostatic potential energy difference between two close points is $\Delta U \sim q\Delta V$. While the potential energy depends on both the point charge q and the charge distribution that generates $\vec{E}$, the electric potential only depends on the source charge distribution. Note that Eq. (2) implies that the maximum value of ΔV occurs for $\theta = 180°$, i.e. when one "moves" along the direction anti-parallel to the electric field. Similarly, the minimum value of ΔV occurs for $\theta = 0°$, i.e. when one "moves" along the direction of the electric field

Equipotential surfaces: Surfaces where V is constant are called "equipotential surfaces". It follows from Eq. (2) that the (infinitesimally close) points "1" and "2" are on the same equipotential surface (i.e., $V_2 = V_1$) if and only if $\theta = 90°$. This implies that the electric field is perpendicular to $\Delta\vec{s}$ and hence, to the equipotential surface containing points “1” and “2”. Therefore, we conclude that equipotential surfaces must be perpendicular to the electric field *lines* at all points.

Calculation of the magnitude and direction of the electric field is difficult in the presence of complicated charge distributions. However, if it is possible to map the equipotential surfaces, then one can obtain information about the electric field based on the fact that the direction of the field at every point is normal to the equipotential surface passing through that point. The magnitude of the electric field can be obtained from the separation of the equipotential surfaces.

In this experiment, you will plot two-dimensional equipotential surfaces (actually lines!) generated by two charged conductors of various shapes. From these equipotential surfaces, you will be able to draw the corresponding electric file lines and estimate the magnitude of the electric field.

Apparatus

Sheets of carbonized paper (each sheet includes a pair of electrodes made of conducting silver ink), Corkboard working surface, Push-pins, three banana wires with clips at one end, Multimeter, power supply, and connecting wires.

Description of Apparatus

Experimental set up for this lab is shown in the figure below in Figure 2.

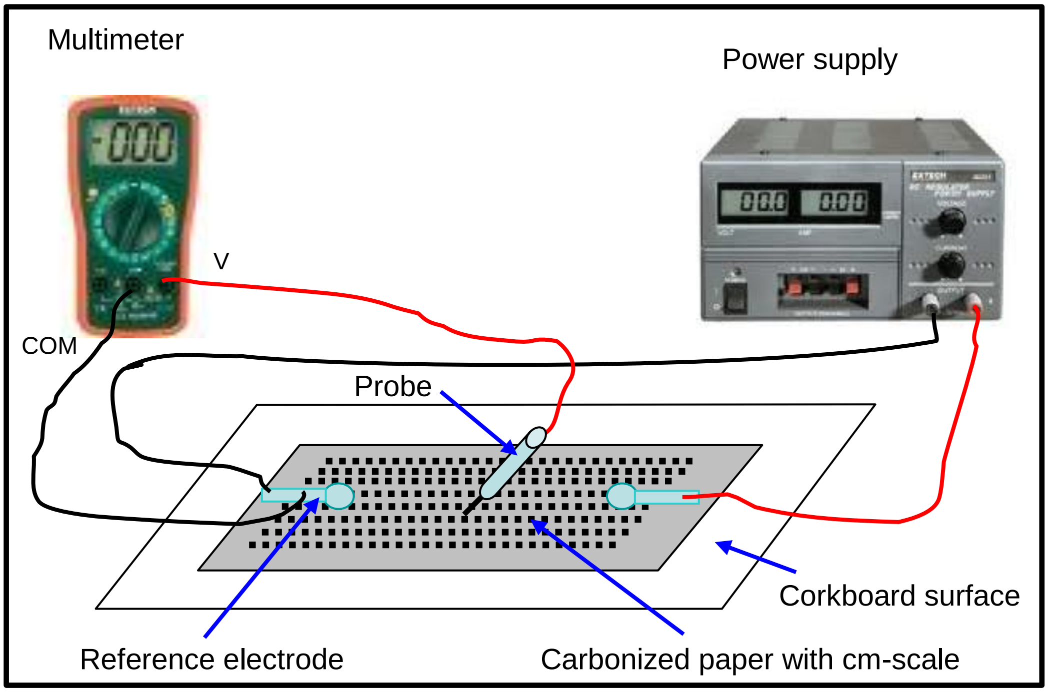


Figure 2: Setup used in the lab. The power supply sets a potential difference between the two electrodes (V = 0 at the reference electrode). The multimeter measures the voltage of the probe (red wire) relative to the reference electrode (black wire). **CAUTION: Make sure the power supply is off before setting up the lab equipment.**

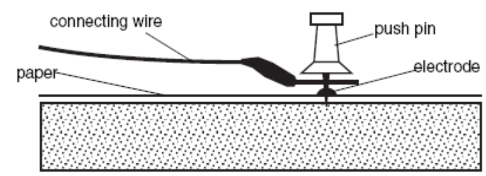
You will use carbonized paper sheets with conductors of different shapes painted using conducting silver ink. There are four different configuration of the electrodes on the carbonized paper sheets in this lab: (a) two point electrodes (b) two parallel electrodes (c) one line and a point electrodes and (d) two line electrodes with a conducting circular area between them. A power supply will be used to create a potential difference between the two conductors. This two-dimensional setup can be used to simulate the patterns of equipotential surfaces and electric field lines that are generated by two metal electrodes in three-dimensions.

At the end of this manual there are graph papers that include a grid similar to the grid printed on the carbonized paper. You will have to measure the potential on the carbonized sheet and ***map the equipotential surfaces on the graph sheet*** attached at the end of this manual. These sheets are your data sheet for this lab. Once you complete plotting the equipotential surfaces, you will be able to draw electric field lines for all four configurations of electrodes.

Procedure

Setting up the apparatus

1. Make sure the power supply is OFF and the "voltage" and "current" control knobs are set at zero position (i.e. turned completely counter clockwise).
2. Use the carbonized paper with two "point" electrodes made of conductive silver ink. Set up the apparatus as shown in Fig. 2. Connect the positive (red) and negative (black) terminals of the power supply to each of the "point electrodes". For proper contact with the electrodes, use push-pins as shown in the figure (in case of wires with a clip at the end, you can simply clip the push pin).



3. Connect a banana plug wire into the COM (where black probe is normally connected) socket on the multimeter. At the opposite end of the wire, insert a clip to easily connect the wire to the electrode, where the negative terminal of the power supply is connected. This is the "reference" electrode, where V = 0. The tip of the red probe from the digital multimeter (plugged into the V-Ω-A input) is used to measure the potential at different points.
4. Set up the multimeter to 20 V (or choose the lowest one that can measure up to 5 V) range by rotating the central knob.

Part I. Mapping of Equipotential surfaces for two "point" electrodes

1. Now turn the power supply on and set the voltage to around 5 V. Turn the current control to allow for a very small current (the red light on the power supply should be off).
2. **Test the circuit if the wire connections are correct:**
   - **a.** Place the tip of the red probe on the black (reference) electrode. What is the reading on the multimeter? Why?
   - **b.** Place the tip of the red probe on the opposite electrode. What is the reading on the multimeter? Why?
3. On one of the graph papers **attached at the end of this lab manual** draw the "point" electrodes at the same location using the scales on graph paper and the carbonized paper.
4. Now poke around with the red probe to find points that are at potential of 4 V. **DO NOT touch the carbonized sheet** with your hand while doing this. Tilt the probe tip a little bit to get stable reading and DO NOT drag the probe tip on the sheet. **Mark the location of these points on your graph paper.**

5. Trace the equipotential line corresponding to 4 V by connecting the points just found. Label the line as 4V.
6. Repeat the previous step for 3.5 V, 3 V, 2.5 V, 2.0 V, 1.5 V and 1.0 V and trace the corresponding equipotential lines. The equipotential lines you obtained are analogous to the equipotential surfaces of two point charges q and –q in three-dimensions.

   Are the equipotential lines as you expected? Is there any two equipotential lines crossing each other?
7. If the equipotential lines are too far apart, you may need to add more equipotential lines for additional voltages (e.g., 1.75 V, 2.75 V, etc). Once you complete plotting your equipotential lines, please **turn off** the power supply.

Part II. Mapping of Equipotential surfaces for two "parallel line" electrodes

1. Remove the previous carbonized sheet and replace by the carbon sheet paper with two **"parallel line" electrodes** made of conducting silver ink.
2. Reconnect the wires as before. Turn on the power supply and test the connection.
   a. Place the tip of the red probe on the black (reference) electrode. What is the reading on the multimeter? Place the probe at different points on the line. Do they read the same voltage on the multimeter?
   b. Place the tip of the red probe on the opposite electrode. What is the reading on the multimeter? Place the probe at different points on the line. Do they read the same voltage on the multimeter?
3. Now, repeat the steps 3 - 7 in Part I. The resulting equipotential lines are analogous to the equipotential surfaces of two parallel, uniformly charged plates with charges *q* and –*q* in three-dimensions.

Part III. Mapping of Equipotential surfaces for one "point" and one "line" electrodes

1. Remove the previous carbonized sheet and replace by the carbon sheet paper with **one "point" electrode and one "line" electrode** made of conducting silver ink.
2. Reconnect the wires as before. Turn on the power supply and test the connection.
3. Now, repeat the steps 3 - 7 in Part I. The resulting equipotential lines are analogous to the equipotential surfaces of one point charge in the presence of a uniformly charged plate with charges *q* and –*q* in three-dimensions.

Part IV. Mapping of Equipotential surfaces for two "line electrodes" with a conducting area between them

In this part of the lab we will repeat Part II but using a slightly modified configuration. In this case, there is a small circular area of conducting ink between the line electrodes. The line electrodes are connected to the power supply, as in Part II, but the small conducting ink area is not connected to the power supply. You will learn how potential and hence electric field behave in the presence of an uncharged conductor placed in an external electric field.

Repeat the steps in Part II. The resulting equipotential lines are analogous to the equipotential surfaces of two parallel and uniformly charged plates, with charges *q* and –*q*, in three-dimensions, in the presence of an uncharged conducting sphere between them.

Computation

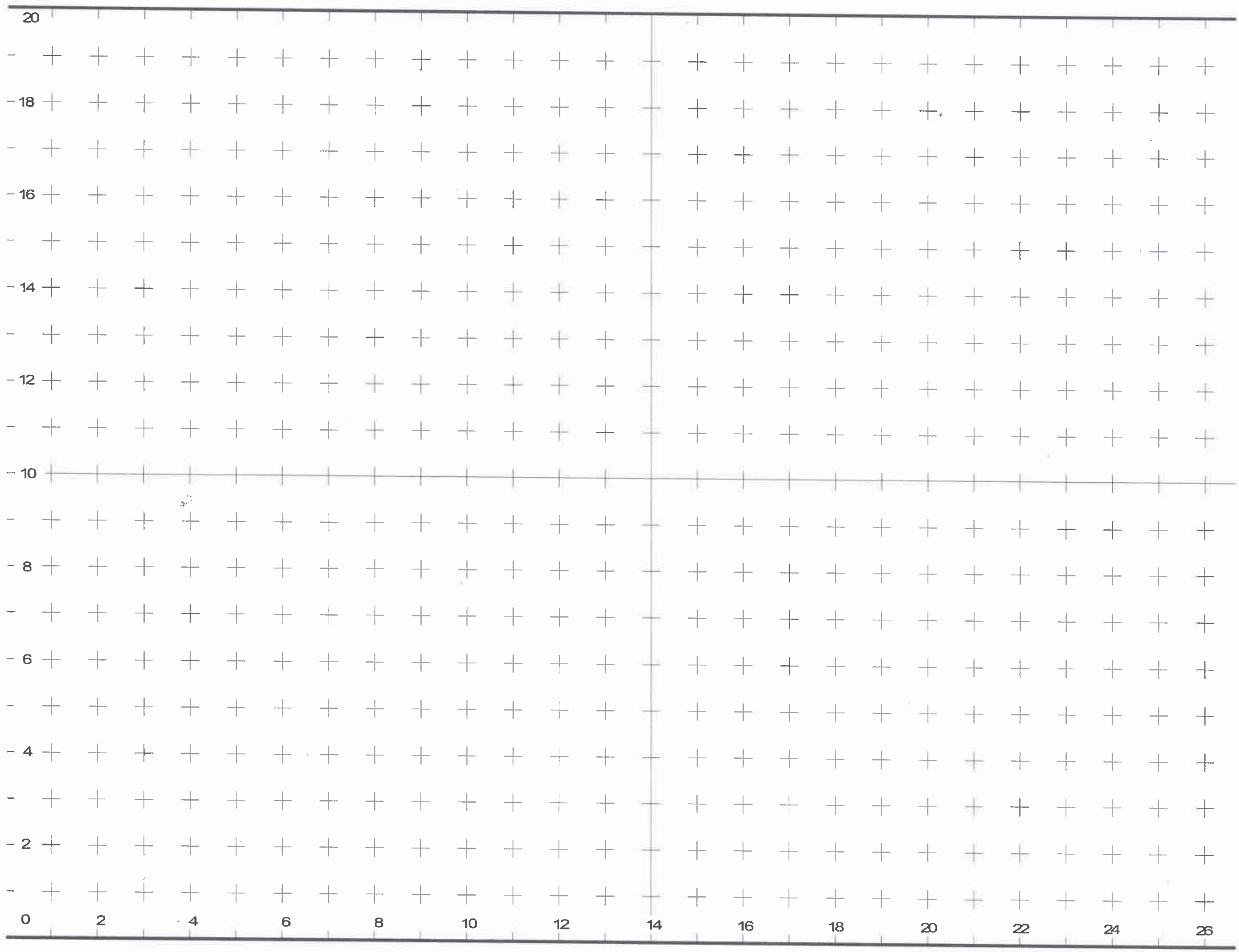
For all **four** configurations of electrodes in the lab, draw electric field lines on the sheets with the equipotential lines you plotted. You may use a different color pen to draw the field lines. Make sure to put arrows on the electric field lines to indicate the direction of the electric field.

Using the values on the equipotential lines and measuring the distance between them, estimate the magnitude of electric field at three different places for each configuration.

Questions

1 How much work is done by the electrostatic force on a point charge that is moved on an equipotential surface (line)? Explain.

2. If the electric field lines are not normal to the equipotential surfaces, what would happen?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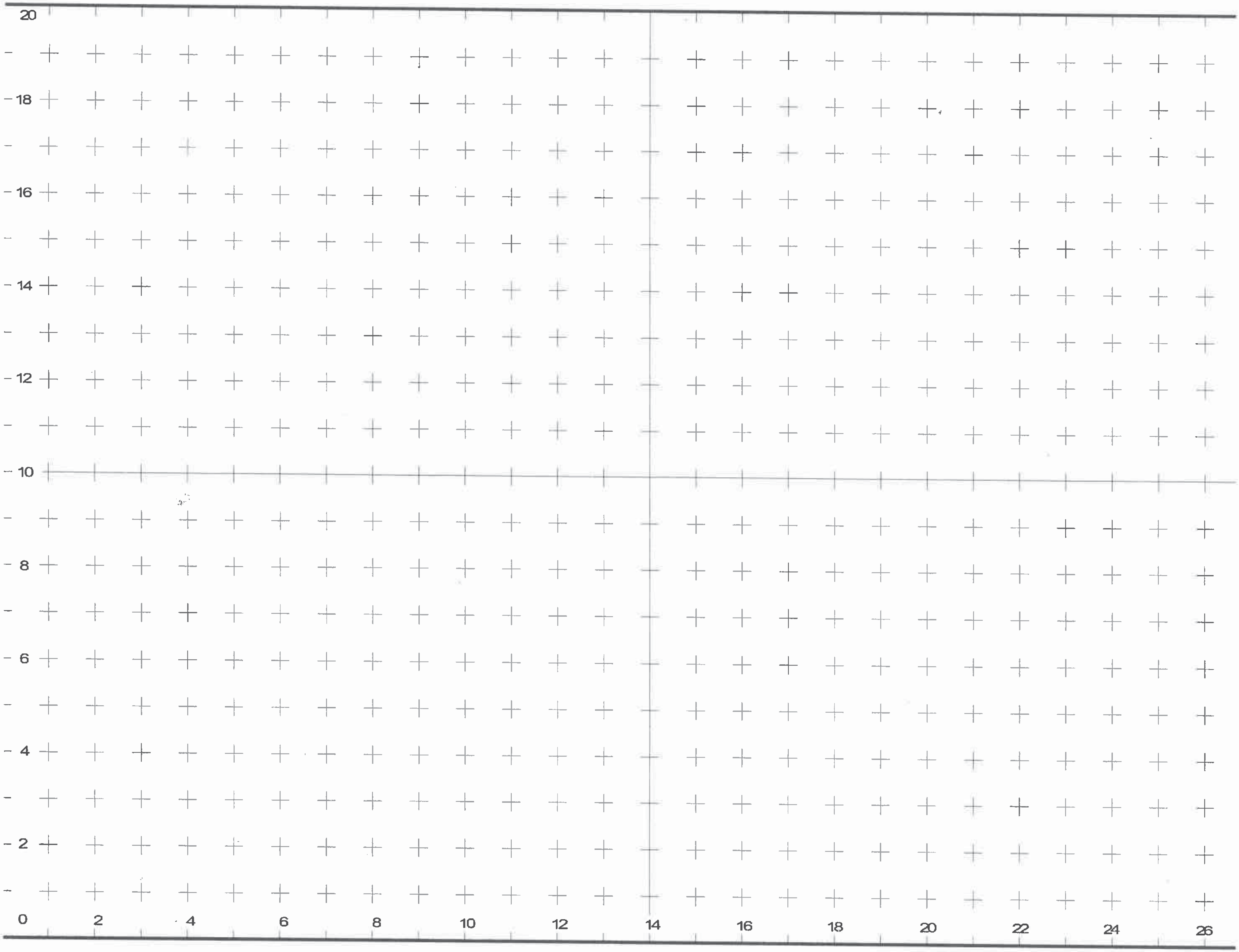
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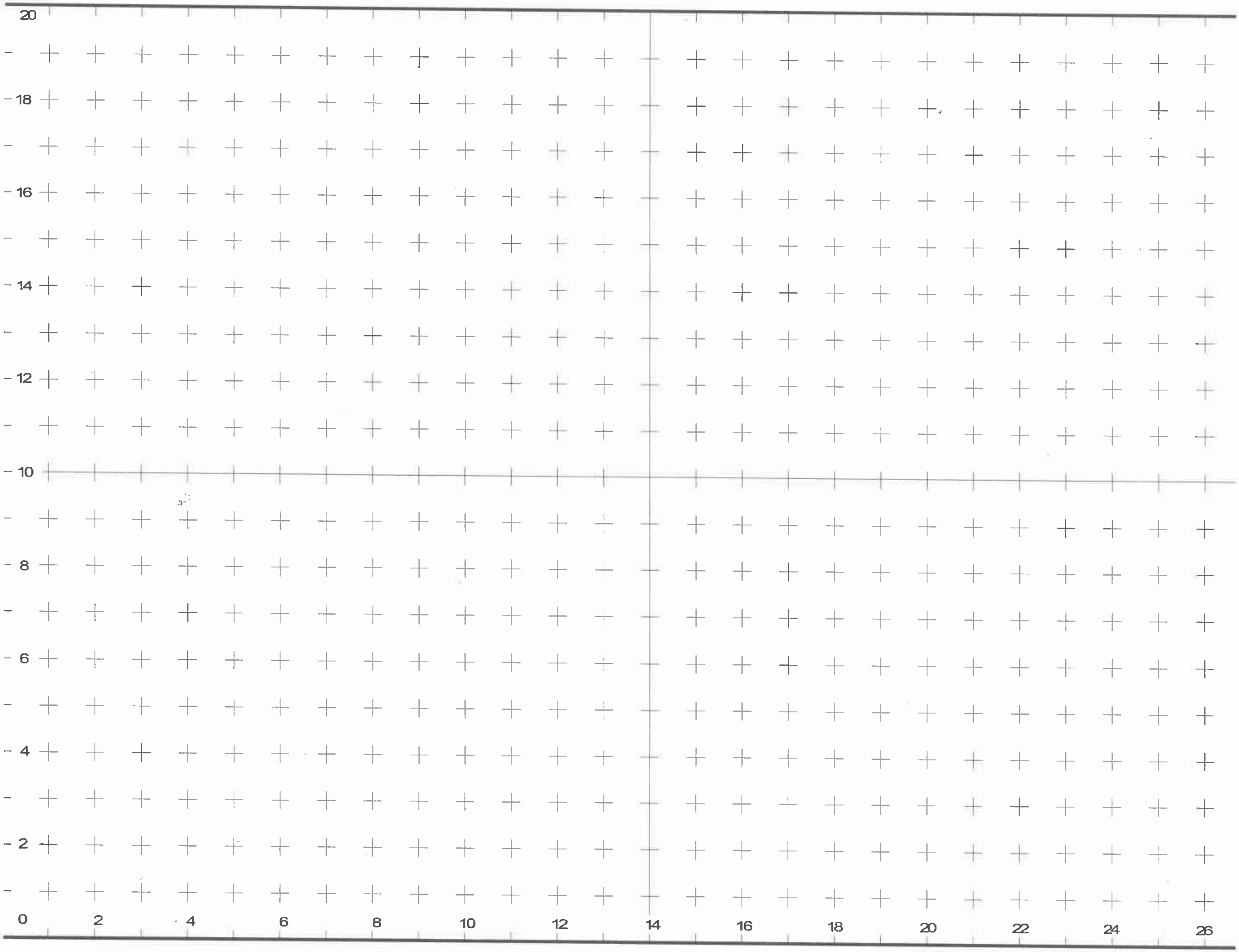
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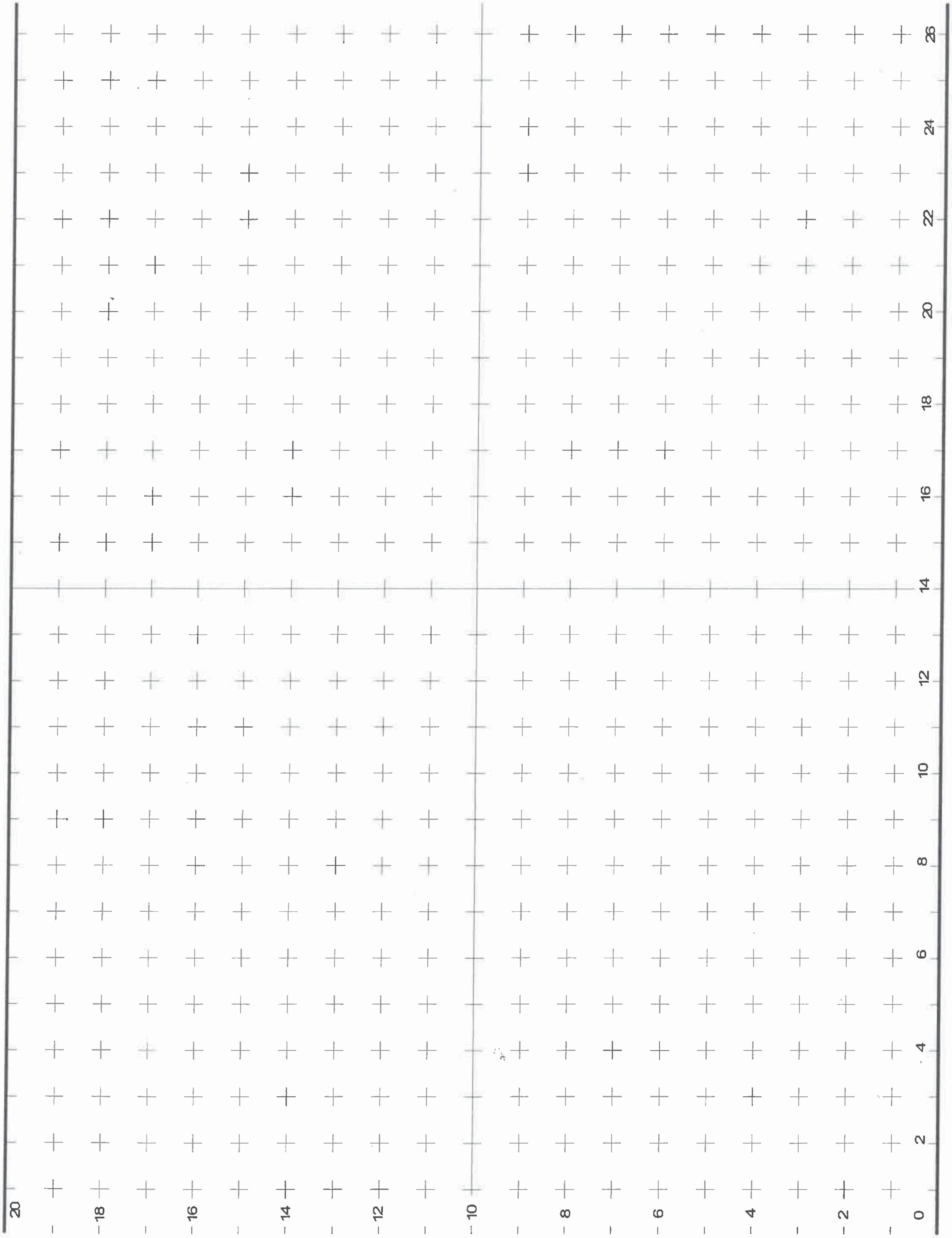
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