Faraday's Law of Induction

Purpose

a. To investigate the emf induced in a coil that is swinging through a magnetic field;

b. To investigate the energy conversion from mechanical energy to electrical energy.

Theory

Part I: Faraday's law of induction

According to Faraday's law of induction, a changing magnetic flux through a coil induces an emf (ε) which is given by

$$\varepsilon = -N\frac{d\Phi}{dt} \quad (1),$$

where $\Phi = B \cdot A$ is the magnetic flux for a magnetic field (B) which is constant over the area (A) and perpendicular to it. Here, N is the number of turns of wire in the coil. In this experiment an induction wand, which is a rigid pendulum with a coil, swings through a permanent magnet. Thus, the area of the coil is constant as it passes into or out of the magnetic field. Therefore, an average induced emf can be written as

$$\varepsilon = -NA\frac{\Delta B}{\Delta t} \quad (2).$$

The negative sign in the above equations is due to Lenz's law. The actual direction and sign of the voltage as the coil enters and leaves the magnetic field is examined in this lab.

Part II: Energy conversion for an induction coil swinging in a magnetic field

To investigate electromagnetic energy conversion, a resistive load is connected to the coil of induction wand which swings in the magnetic field. In a resistive load, electrical power is dissipated as heat. The power dissipated in the resistor is calculated by measuring the voltage across the load resistor. The energy converted to thermal energy is determined from the power versus time graph. This energy is compared to the loss of potential energy determined from the amplitude of the pendulum.

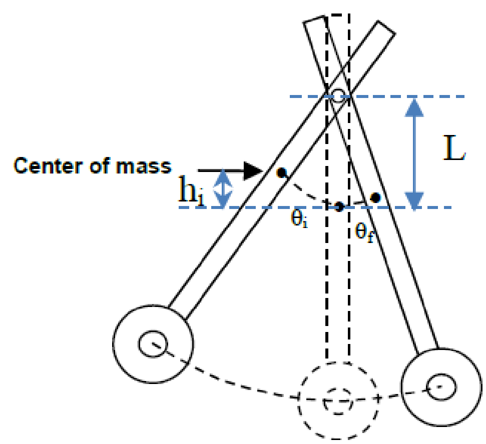

Figure 1: Oscillating wand with a coil in a magnetic field. Conversion of mechanical to electrical energy.

Figure 1 shows a swinging induction wand. When the coil in the induction wand swings in the magnetic field, a current is induced in the coil. Let L be the distance from the axis of rotation to the center of mass. If the center of mass of the pendulum starts from rest at a height h, its potential energy is

$$U = mgh = mgL(1-\cos\theta) \quad (3)$$

Here, the height h is measured from the lowest position of the center of mass when it swings (angle $\theta = 0$, see **Fig. 1**).

If the initial and final heights are h_i and h_f respectively, then the decrease of potential energy is

$$\Delta U = mg\,(h_i - h_f) \qquad (4)$$

where $h_i = L - L\cos\theta_i$ and $h_f = L - L\cos\theta_f$

According to the conservation of energy, the decrease in potential energy, ΔU, will be equal to the **energy lost to the friction, plus the energy converted into electrical energy**. **We will examine these two forms of energy conversion separately.** We first examine the energy lost due to friction alone by disconnecting the load resistor in the circuit and measuring the decrease of potential energy as given in Eq. 4.

For energy converted into electrical energy, we will connect a load resistor with coil in the wand. When a resistive load (R) is connected in the circuit (**Figure 2)**, the emf induced in the circuit, ε is given by

$$\varepsilon = IR + Ir \qquad (5)$$

where r is the internal resistance (the resistance of the coil itself), R is the resistance of a load resistor connected in the induction wand and I is the current in the circuit. Note that R and r are in series in the equivalent circuit. For a resistive load, electrical energy is dissipated as heat in the circuit. The power in the circuit is given by

$$P = I\varepsilon$$

$$P = I^2(R + r)$$

$$P = \left(\frac{V}{R}\right)^2 (R + r) \qquad (6),$$

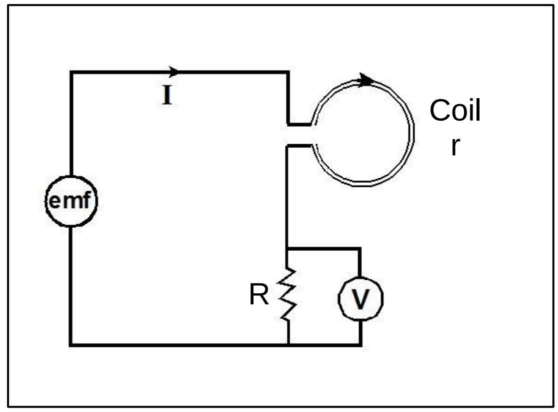

Figure 2: Equivalent circuit of the set up. The coil has a resistance R while the voltage is measured across a "load resistor", r that is mounted across the banana plugs.

here, V is the voltage across the external resistor (R),

V = IR. For a given R and r, power versus time can be determined by measuring voltage across the load resistor (V) as function of time. The thermal energy dissipated in the resistors is given by

E = ∫Pdt = the area under a graph of power versus time.

Apparatus

Induction coil wand, rotary motion sensor, lab stand, variable gap magnet, banana plug with load resistor, multimeter, balance, ruler, LabQuest interface, computer with Logger Pro software.

Description of Apparatus

We will use PASCO Faraday's experiment apparatus for this lab. The figure below shows the major apparatus used in this lab and a typical set up. Descriptions of the major components of

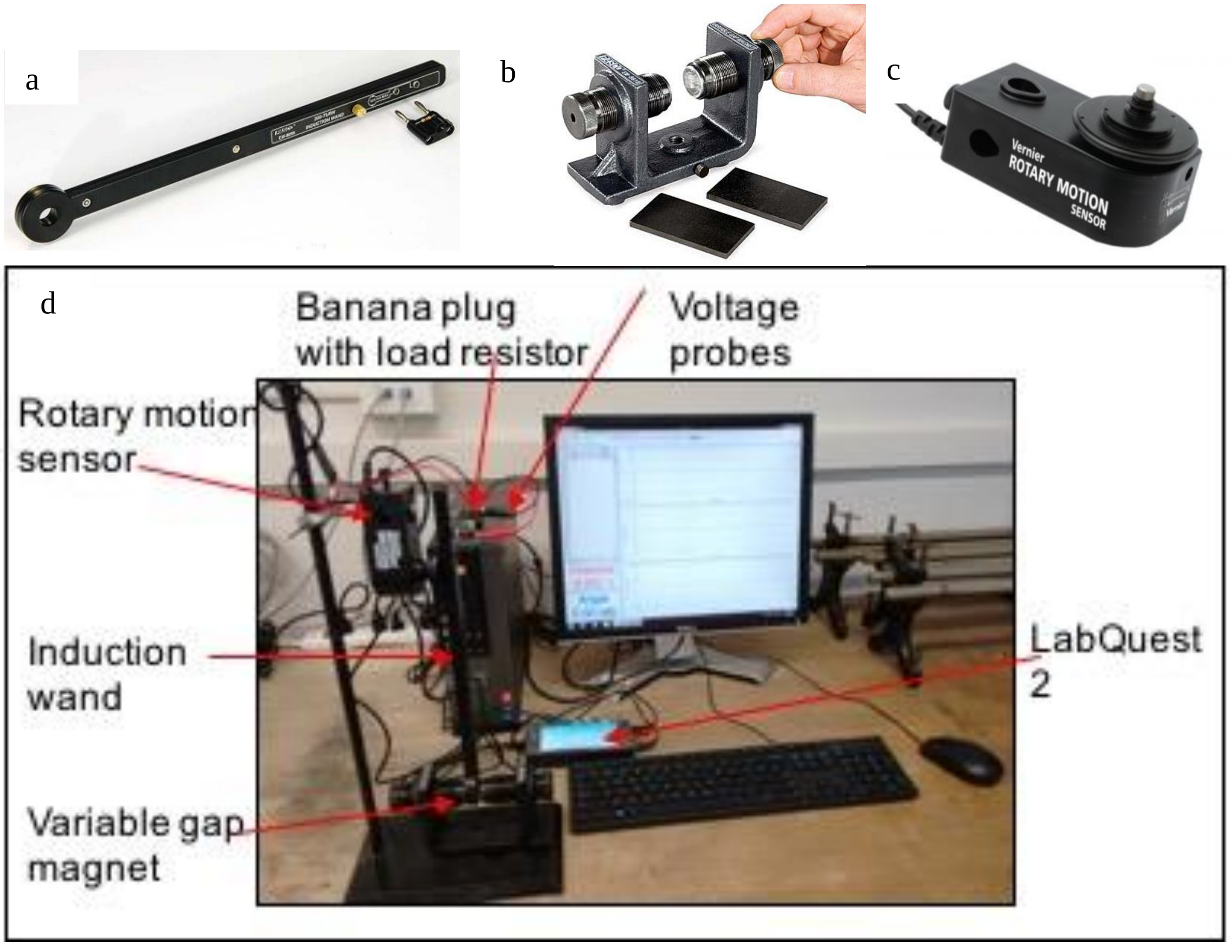

Figure 3: (a) Induction wand and banana plug resistor, (b) Variable gap magnet, (c) Rotary motion sensor and (d) typical experimental setup.

apparatus for this lab are now given.

The Induction Wand is a rigid pendulum with a coil at the bottom end, and coil ends are connected to the banana terminals at the other end. It has 200 numbers of turns of coils made by copper. The coil has inner diameter of 1.9 cm and outer diameter of 3.1 cm. The banana terminal at the top end is used to insert a load resistor connected across a banana plug (also shown in Fig. 3a). A through-hole in the wand allows the pendulum to be connected to a Rotary Motion Sensor.

The Variable gap magnet has two one-inch diameter neodymium magnets mounted on a heavy-duty cast iron base. The gap may be varied from 0.5 cm to 8.9 cm using the adjustment screws. Two flat pole pieces may be used to provide a uniform magnetic field. Magnetic field between the poles also varies with the gap. The magnetic field in the gap is around 0.30 T when the gap between the poles is about 2 cm.

The Rotary motion sensor is used to attach the induction wand to swing and to measure the angular position of the wand. When the wand is screwed to the rotary motion sensor, the axle of the sensor also serves as pivot for the oscillation.

In this lab, you will study Faraday's Law of induction using a wand with coil which swings through a magnetic field produced by the variable gap magnet. You will also examine conversion of mechanic energy into electrical energy by comparing the energy dissipated in load resistors to the loss of mechanical energy of the pendulum wand. A voltage sensor is used to measure the emf induced (or voltage across the load resistor). Both sensors will be connected to a Vernier LabQuest interface device and then to a computer. Logger Pro software is used for the measurement and analysis of the data. The Logger Pro software will plot graphs of the voltage and the angle of the induction wand versus time. The power dissipated in the resistor is calculated from the graph of voltage versus time across the load resistor. The energy thereby converted to thermal energy is determined from the area under the power versus time graph.

Procedure

Initial measurement

Before starting the experiment, perform the following starting measurements and record in the data sheet:

1. Unscrew the wand if it was attached. Plug in the banana plug resistor at the top end of the wand and try to balance it to find its center mass. Now, measure the distance (L) from the pivot point to the center of mass (see Figure 1).
2. Measured the mass of the wand (m) including the banana plug with a resistor. There will be a shared lab balance available for this purpose.
3. Unplug the banana plug resistor from the wand and measure the resistance of the coil of the induction wand (r) and the resistance of the load resistor attached in the banana plug (R). Use the multimeter provided for this purpose.
4. Find the number of turns (N) in the coil and its area (A) from the specification of the induction wand (some information are given in the section of description of apparatus).

Part I: Induced emf by moving coil

In this part, the induction wand is swung into the magnetic field. **We are not using banana plug resistor in this part**. The setup for this part is shown in the Figure 4.

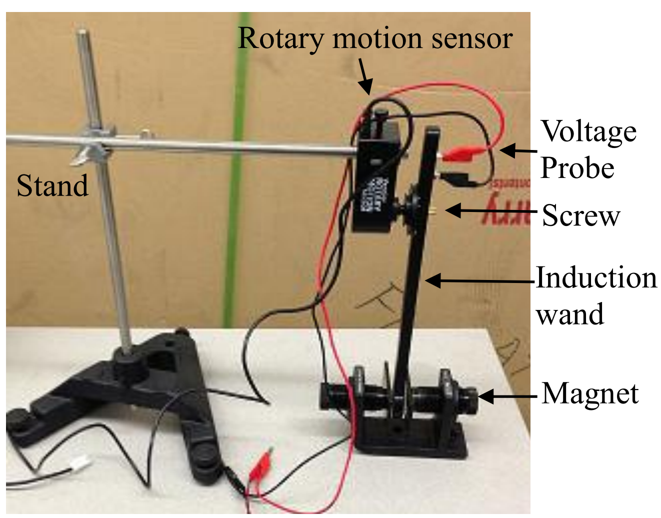

Fig. 4. Experimental set up for measuring induced emf in a coil.

1. Assemble the Rotary Motion Sensor with the stand. Attach the induction coil wand to the Rotary Motion Sensor with a screw.
2. Put the variable gap magnet to the lower end of the wand. Put the gap between the magnet poles about 2 cm. Adjust the height of the stand so that the coil is in the middle of the variable gap magnet. Make sure the wand can always swing through the magnet without hitting it.
3. Insert terminals of the voltage probe to the induction wand. You may use two banana cables to connect

between the voltage probe and the wand if the probe wire is too short. Drape the wires over the rods as shown in Figure 4 so that the wires do not exert any torque on the coil as it swings.

4. If not already connected, plug the voltage probe into the analog input of the LabQuest interface and the Rotary Motion Sensor into a digital input of the LabQuest interface. (The digital inputs to the LabQuest2 interface are to be found under a small flap.) The LabQuest should be connected to a computer.

5. Now, Open Logger Pro in your computer. Set samplings rate to 200 samples/sec and collection time to 20 sec. [Click *'Experiment'* and then *'Data Collection'* in the menu bar to change the setting]. Before taking the data, zero the sensors. While the wand is at rest in its equilibrium position between the magnets and shorting the terminals of voltage sensor, click . You should see zero or very close to zero values on the computer.

6. Now you are ready for measurement. Click *'Collect'* in the menu bar. **Do not move the wand yet!** Once you see the green *"collect"* button on the menu bar turned into red *"stop"* and the Angle reading is zero on the screen, (see Fig. 5), rotate the wand to an angle (For example $\theta_i = 0.4$ rad) and let it go. Click *'STOP'* after a few swings. You should get two graphs, one for voltage versus time and another for angular position versus time. If necessary, adjust the scales for proper viewing of the graphs.

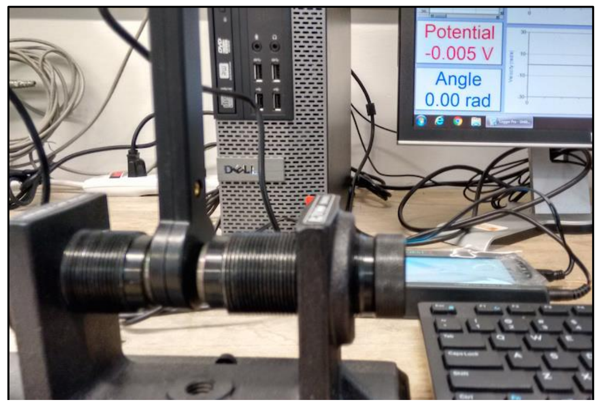

Fig. 5. Initial position of the wand before recording the data. Note the Angle registered as zero in Logger Pro software.

7. If the first run did not go well, repeat the previous step. Look at the graphs carefully.
   a. Voltage measured by the voltage sensor is the emf induced in the circuit. Why?
   b. Why is the sign of the emf of the second peak opposite to the sign of the first peak?
   c. Comparing time scales of voltage graph and angular position graph, identify the positions of the coil when it is entering the magnet and when it is leaving the magnet.
   d. Why is the emf nearly zero when the coil is passing through the center of the magnet?

8. Highlight the first induced voltage region and determine the average voltage using *"Statistics"* option under *"Analyze"* in the menu bar. Write down the average voltage of the first peak in your data sheet. Explain the answers to the questions above and include the graphs of emf and angle vs time (with the same time scale) in your lab report.

Estimation of induced emf

You have to estimate the induced emf using equation (2) and compare with the experimental result. In order to estimate the time (Δt) for which the flux is changing while the coil is entered into the magnetic field examine the voltage versus time graph carefully. Determine the difference of the times from the beginning to the end of the first peak and record in the data sheet. You can highlight and *"zoom in"* the area and use *"examine"* option on the menu bar to find these values. You will need this value for the computation.

Part II: Mechanical to Electrical Energy Conversion

The apparatus setup for this part is shown in the Figure 6 below. The setup is very similar to Part I but we attach the banana plug to the induction wand in this part. First, you will measure the change in potential energy in open circuit condition to determine the energy lost due to friction. After that you will measure the change in potential energy and voltage across the load resistor in a closed circuit condition to determine change in potential energy and the electrical energy converted into the circuit.

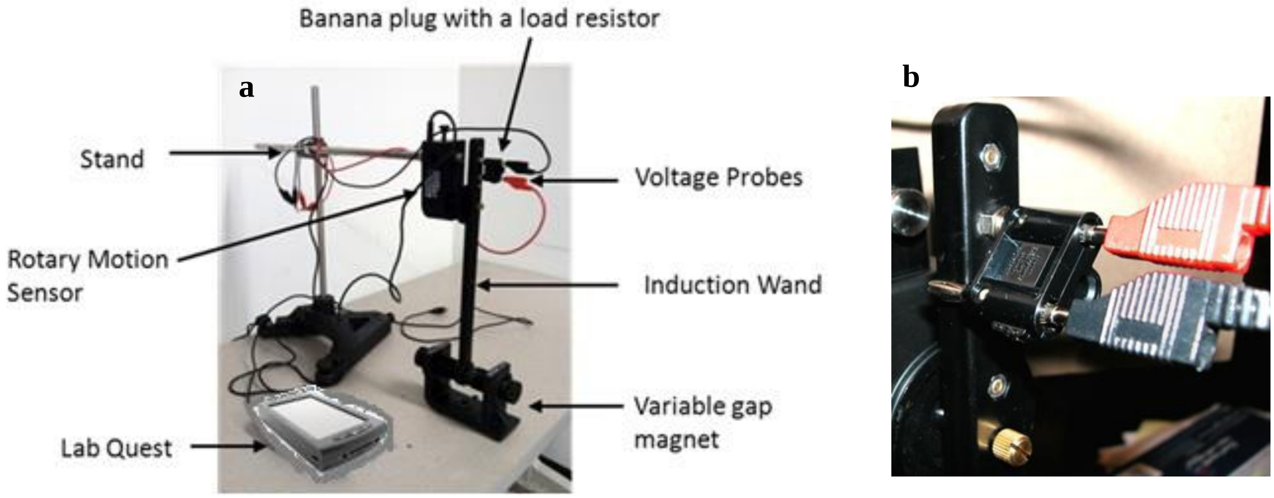

Fig. 6. (a) Experimental set up. Banana plug resistor is used as a load. (b) Banana plug is disconnected to measure the energy lost due to friction.

(a) Energy lost due to Friction alone

1. Adjust the gap between the magnet poles. Put the magnet poles as close together as possible but the wand should swing through the magnet without hitting it. Adjust the height of the wand if necessary. The center of the coil should be at the middle of the gap when the wand is at rest.
2. Remove the voltage probe connectors from the wand and plug the load resistor with one of its plugs out to the side and one plug in the wand (see Figure 6b). It makes the circuit open without changing the center of mass. Connect the voltage probe terminals across the banana plug resistor (make sure the circuit is still open!).
3. Make sure the voltage sensor and the Rotary Motion Sensor probe are still connected properly into the LabQuest interface which is then connected to the computer.
4. Before collecting the data, make sure the samplings rate is 200 samples/sec and collection time is 20 sec. If not adjust them as you did before in part I.

Measurement and Analysis

5. Before taking the data, zero the sensors as before. While the wand is at rest in its equilibrium position and shorting the terminals of voltage sensor, click on the menu bar.
6. Now reconnect the voltage probe and click *'Collect'* with the coil at rest in its equilibrium position between the magnets. **Do not move the wand yet!** Once you see the green "collect" option on the menu bar turned into red *"stop"* and the Angle reading on the screen is zero

(see Fig. 5), rotate the wand to an initial angle. (For example $\theta_i = 0.4$ rad) and let it go. Click *'Stop'* after it has swung to the other side.

7. You should have two graphs as in Part I. Change the axis scales if necessary. Repeat the previous step if the first trial did not go well.
8. On the graph of angle vs time, determine the angle to which the pendulum rises after it passes once through the magnet.

Potential Energy (U) versus time graph

You can display a graph of the potential energy versus time from the data of angle you just measured from the rotary motion sensor. Follow the instructions given below

9. On the computer, click *"New Calculated Column'* under *'Data'* on menu bar. It will open a new dialogue box.
   - Name it 'Potential Energy' and U as short name, and 'Joules' for unit.
   - Under the expression, type the right hand side expression of Eq. (3) as in any computer language. You will need to use the * symbol for multiplication. You may directly use the values for m, g, and L or use them as parameters and then substitute the values.
   - For angle, click 'Variables (Columns)' and choose θ (angle). Click 'Done' to complete.

   This will add a column for you with calculated potential energy values corresponding to the angles measured by the rotary motion sensor.
10. Insert a new graph (Click on the "Insert" drop-down menu) to display U versus t graph.
11. Examine the graph of U versus t graph to calculate the decrease in potential energy, which is the energy lost due to friction $E_{friction}$, during one swing and record the value in the data sheet. Save and include this graph in your report.

Change in potential energy and Energy converted into Electrical energy

(b) Change in potential energy (ΔU_{total})

1. Now, insert both plugs of the load resistor (R) in the wand. This completes the series circuit of the load resistor and coil.
2. Repeat the previous steps 5 through 10. You should rotate to the same initial angle (θ_i) and let it go. Click *'STOP'* after it has swung to the other side.
3. Measure the angle to which the pendulum rises after it passes once through the magnet. Note the initial and final angles. Determine the change in potential energy from the U versus t graph and record in the data sheet. You also need to include this graph in your report. This is the total change in potential energy, ΔU_{total}.

(c) Energy converted into electrical energy

4. In order to measure the electrical energy converted into the circuit, add a *'New Calculated Column'* to generate Power (P) versus time (t) graph (Similar to the U versus t graph before). **This time you are using Eq. 6**. *'Insert'* a new graph of Power versus time. How can you get energy from the graph of power vs time?
5. In the Power versus time graph, use your mouse to highlight *both peaks* that correspond to the swing of the wand and find the area (integral value). (This can be done by using the *"Analyze"* drop-down option). This area is the electrical energy in the circuit of the heat dissipated by the resistors, $E_{electrical}$. Save this graph for your report and record the data in the data sheet.

Computation

For Part I: Estimation of emf induced

- What is ΔB in Equation 2, if we assume that the magnetic field in the induction coil is changed from 0 to B (the values of B is given in the section of description of apparatus)?
- Calculate the average area of the coil from the specification of the coil (values given in the description of apparatus section).
- Using Faraday's law (Eq. 2) estimate the average emf induced.

Compare your result with experimental value. What are the possible source of errors in the calculation and experiment?

For Part II: Energy conversion

Add the energy dissipated by the resistor ($E_{electrical}$) and the energy lost to friction ($E_{friction}$). Compare this to the total change in potential energy of the pendulum, ΔU_{total}.

Questions

1. Why is the sign of the second voltage peak opposite to the sign of the first peak in voltage versus time graphs?
2. Why is the voltage zero when the coil is passing through the exact center of the magnet?
3. With a load resistor connected to the induction wand, why is the second peak voltage smaller than the first one in the voltage versus time graph? Is this voltage emf?
4. Why does the wand slow down significantly when a load resistor is connected?
5. Why do you need to include both peaks in the integration of power versus time graph to determine energy?
6. What are the possible errors in this experiment?

Data Sheet

Date experiment performed:

Name of the group members:

Initial measurement

Center of mass, L =

Mass of wand with banana plug resistor, m:

Resistance of coil, r =

Resistance of load resistor, R =

Number of turn, N =

Inner diameter of the coil, d_i =

Outer diameter of the coil, d_o =

Magnetic field, B =

Part I: Induced emf

From first peak of voltage vs time graph,

Average induced emf from voltage vs time graph, $\mathcal{E}_{expt}$ =

Initial time the coil enters the region of magnetic field, t_1 =

The time the coil is completely inside the region of magnetic field, t_2 =

$\Delta t = t_2 - t_1$ =

ΔB =

Average area of the coil (from diameter values provided)

Average estimated ε =

Part II: Energy conversion

(a) Loss of energy due to friction

Starting angle (in rad) =

Finishing angle on other end (in rad) =

Energy lost due to friction, $\Delta U_{friction}$ found from graph of U vs t =

(b) Total decrease of potential energy

Starting angle (in rad) =

Finishing angle (in rad) =

Total energy lost, ΔU_{total} found from graph of U vs t =

(c) Energy dissipated in the circuit

$E_{electrical}$ found by integrating P vs t graph =

$E_{friction} + E_{electrical}$ =