RC Circuits

Purpose

a. To study the transient behavior of voltage and current in RC circuits.

b. To measure an RC circuit time constant (τ), $\tau = RC$.

c. To determine the capacitance of an unknown capacitor from the time constant.

d. To verify the equivalent capacitance of capacitors in parallel and series combinations.

Theory

The voltage drop across a capacitor of capacitance C is given by

$$V_C = \frac{Q}{C} \tag{1}$$

The voltage drop across a resistor of resistance R is given by

$$V_R = IR \tag{2}$$

In this laboratory you will measure the voltage across the resistor and capacitor in two situations (1) when the capacitor is being charged and (2) when the capacitor is being discharged.

Figure 1 shows the circuit of charging and discharging of a capacitor C. When the single pole double throw switch (SPDT) is toggled to position 1, it is a charging circuit as it completes the circuit with the battery, resistor and capacitor. When the switch is toggled to position 2, it disconnects with battery and makes a discharging circuit.

Draw separate circuit diagrams for charging and discharging processes on your note book. Indicate clearly the direction of current and the polarity of voltage across the capacitor.

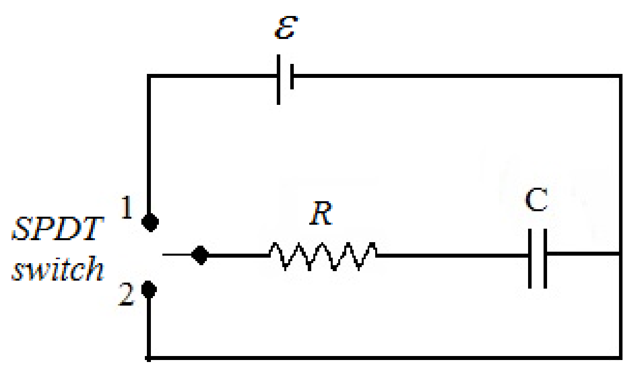

Figure 1: Circuit diagram for charging and discharging of a capacitor.

Charging

Referring to the Fig. 1, when the capacitor of capacitance C is being charged by the power supply of emf, $\mathcal{E}$, through the resistance R, the voltages around the loop satisfy:

$$\mathcal{E} - V_C(t) - V_R(t) = 0$$

$$\mathcal{E} - V_C(t) - R\,I(t) = 0 \tag{3}$$

The solutions to the above expression for $V_C(t)$ and $I(t)$ are the time-dependent (transient) behavior of voltage across the capacitor and current in the circuit. If the capacitor is uncharged in the beginning, the solutions to the above expression for $V_C(t)$ and $I(t)$ given by

$$V_C(t) = \mathcal{E}\,[1 - e^{-t/\tau}] \qquad \text{(Inverse exponent)} \tag{4}$$

$$I(t) = \frac{\mathcal{E}}{R}[e^{-t/\tau}] = I_0[e^{-t/\tau}] \qquad \text{(Natural exponent)} \qquad (5)$$

where τ is called the time constant of the circuit and given by

$$\tau = RC \qquad (6)$$

and $Io = \mathcal{E}/R$ is the maximum current in the circuit. When the capacitor is fully charged, the charge on it Q = C/ε.

Discharging

When the capacitor is being discharged through the resistance R, by toggling the SPDT switch to position 2, the transient behavior of $V_C(t)$ and $I(t)$ are given by

$$V_C(t) = V_0\,[e^{-t/\tau}] \qquad (7)$$

$$I(t) = -I_0[e^{-t/\tau}] \qquad (8)$$

In the above expressions $V_o = Q/C$, and $\tau = RC$.

Apparatus

Two capacitors, power supply, decade resistor box, wires, differential voltage probe, current probe, LabQuest interface, and a single pole double throw switch (SPDT).
Note that the capacitors are polarity sensitive and you should wire the capacitors carefully (if you are unsure about how to wire the capacitors ask your instructor for help).

Description of Apparatus

In order to realize the circuit shown in the Fig. 1, we will use a power supply instead of the battery. We will use a differential voltage probe for voltage measurement and current probe for current measurement. We will use a decade resistance box so that we can vary the resistance in the circuit. Schematic of the circuit connection that you will construct will look like in Fig. 2.

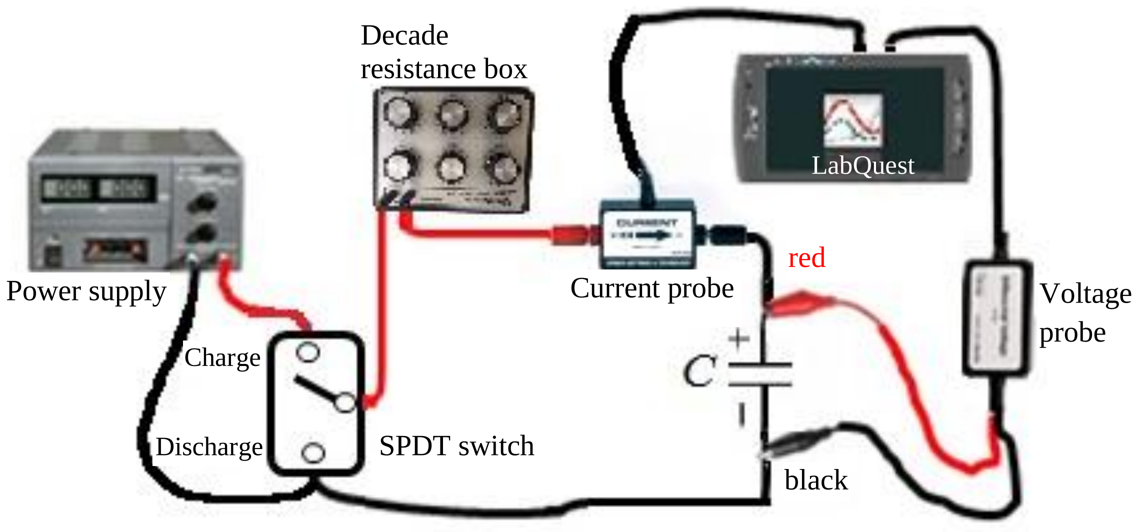

Fig. 2. Apparatus and schematic of circuit connection.

Procedure

1. Make sure the power supply off. Wire the circuit as shown in the figure above using one of the capacitors given. Note that the capacitor has a polarity - connect it properly! Connect voltage probe across the capacitor and connect the current probe in series with the resistor and capacitor using the proper polarities. The sensors should be connected to a LabQuest which is then connected to a computer.
2. Open Logger Pro in the computer. Before taking the measurements, zero the sensors by clicking  on menu bar (or from set up sensor) **with the power supply off**. Set the sampling rate to 100/ sec {*Experiment →Data collection→ Sampleing Rate = 100→ Done*}. Now slowly rotate the voltage and current knobs and set the voltage to 5 V.
3. Make sure that the capacitor is initially discharged by placing the switch in the discharge position, with R set to zero. Then set the resistance, using the decade box, to a suitable value (compute an approximate value of R to give $\tau \sim 0.5$ sec.). Note down the value of resistance in table 1.
4. Now, click ***"Collect"*** on the computer screen and then, shortly after the data starts collecting, throw the switch to the CHARGE position. Wait until the capacitor is fully charged and throw the switch back to the DISCHARGE position. You will now have, in one graph, both the charging and discharging curves for the voltage and current. **Save the data** in a temporary file before continuing!. Change the scales in both graphs to clearly see the plots on the screen. You may use Auto Scale option by clicking 
5. **Curve Fitting**: For both the charging and discharging portions of the curves for the capacitor, you will select an appropriate region and fit the data with the appropriate function to determine the time constant (τ) for the circuit. First, select a portion of a graph in charging portion from voltage versus time graph for fitting. The region you select should begin and end on the portion of the curve that is changing significantly. After selecting the region, now click on "***Analyze***" and select "**Curve Fit**", and then choose the function "Inverse Exponent" or "Natural Exponent" depending upon the nature of the curve. Then click on "Try Fit" and finally on "OK" to show the fitting parameters in a box on the graph. **Note that τ is the reciprocal of the fitting constant C** (this C is not the capacitance!). You will also see bracket symbols ([ ]) in the beginning and end of the graph covering the data used to make fitting. You can move those brackets to modify the portion of the graph.

   Fit the curve for the discharging portions too. If the two values of C are in good agreement and if the "Root MSE" for each is less than ~ 0.01, record the average values of τ. Otherwise repeat the analysis or entire measurement.

   Complete the fitting for the graph from current versus time too. Record the values in table 2.

   You should include a graph from ONE observation in your report. Print **ONLY ONE** of your favorite graph.
6. For this first measurement only, examine the table of values for $V_C(t)$ and I(t) for five different values of t spanning the time during which (i) the capacitor is charging and (ii) the capacitor is discharging, compute the value of $[V_C(t)+V_R(t)]$ and record these values. Here, $V_R(t) = I(t).\ R$.

   [Physics 2150 only] Using the "Integrate" function in the analysis software, integrate I(t) separately for both the charging and the discharging processes. Record the values of these integrals and also record the voltage V_C on the fully charged capacitor.

7. Repeat steps 4 - 5 above for at least three other significantly different values of R. These should span the entire range for which you can measure the time constants, i.e. from $\tau \sim 0.1$ sec to $\tau \sim 2$ sec. You will now have at least four values of τ, corresponding to four different values of R, which can be used to construct a graph of τ vs. R.
8. Repeat the procedure described in steps 4 - 6 for another capacitor, thereby obtaining at least four values of τ as a function of R for the second capacitor.
9. Now, connect the two capacitors in series and measure the time constant using the method described in 3 - 5 above, for one value of R only.
10. Connect the two capacitors in parallel and measure the time constant using the method described in 3 - 5 above, for one value of R only.

Computations

In this laboratory you have observed the charging and discharging behavior of a RC circuit. And measured the time constant ($\tau = RC$) for the circuit.

Compare the nature of the voltage versus time graph and current versus time graph while charging and discharging.

For capacitor 1, plot a graph of τ vs. R from the data in table 1 and 2. Determine the experimental value of the capacitance (C_1) from the graph and record in table 3. Compare your result with the capacitance value written on the capacitor.

Similarly, for capacitor 2, plot a graph of τ vs. R from the data in table 1 and 2. Determine the experimental value of the capacitance (C_2) from the graph and tabulate in table 3. Compare your result with the capacitance value written on the capacitor.

Calculate the experimental value of capacitance from the value of τ for series combination of the capacitors and tabulate in table 3.

Similarly, calculate the experimental value of capacitance from the value of τ for parallel combination of the capacitors and tabulate in table 2.

Calculate the theoretical values of capacitance for series and parallel combinations. Theoretical values are computed using the appropriate formulas for series and parallel combinations of capacitors and the experimentally determined values of C for the individual capacitors. What are the formulae for calculating equivalent capacitance in series and parallel combination?

[Physics 2150 only] Compute and compare the values of Q determined by (i) $Q = \int I\, dt$ (ii) $Q = C\, V_C$ for both the charging and discharging processes in step 6.

Questions to be answered in your report:

1. Does your graph of τ vs. R pass through the origin? Should the graph necessarily pass through the origin? What does a non-zero intercept signify?
2. Discuss your observations of $[V_C(t) + V_R(t)]$ for both the charging and discharging processes.
3. In one of the current versus time graph, choose a current (near the time ~ 0 sec), say I_1. Now find the current which is 1/e (= 0.37) of I_1. How long does it take to change from I_1 to 0.37 I_1? Compare it with time constant.

4. [Physics 2150 only] Compare and discuss the values of Q determined by the two different methods in part 5).
5. Do your experimental values for the series and parallel combinations of capacitors agree with the theoretically calculated values? Discuss in terms of the experimental errors involved.
6. Your capacitors were charged to a maximum voltage of 5 volts. In each of the four cases (that is for the individual capacitors and for the series and parallel combinations), when the capacitors were fully charged, what was:
   A. The charge (in microcoulombs) stored on the plates of each capacitor.
   B. The energy (in microjoules) stored in each capacitor.

Data Sheet

Date experiment performed:

Name of the group members:

Table 1. For voltage versus time graph

Capacitance, C	**Resistance (Ω)**	**Charging (τ)**	**Discharging (τ)**	**Average (τ)**
$\mathbf{C_1}$				
$\mathbf{C_1}$				
$\mathbf{C_1}$				
$\mathbf{C_1}$				
$\mathbf{C_2}$				
$\mathbf{C_2}$				
$\mathbf{C_2}$				
$\mathbf{C_2}$				
$\mathbf{C_{eq}}$ **- Series**				
$\mathbf{C_{eq}}$ **- Parallel**				

Table 2. For current versus time graph

Capacitance, C	**Resistance (Ω)**	**Charging (τ)**	**Discharging (τ)**	**Average (τ)**
$\mathbf{C_1}$				
$\mathbf{C_1}$				
$\mathbf{C_1}$				
$\mathbf{C_1}$				
$\mathbf{C_2}$				
$\mathbf{C_2}$				
$\mathbf{C_2}$				
$\mathbf{C_2}$				
$\mathbf{C_{eq}}$ **- Series**				
$\mathbf{C_{eq}}$ **- Parallel**				

Table 3

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_{eq}}$ **- Series**	$\mathbf{C_{eq}}$ **- Parallel**
Experimental				
Theoretical	**--**	**--**		