The Series RLC Circuit and Resonance

Purpose

a. To study the behavior of a series RLC circuit in an AC current.

b. To measure the values of the L and C using the impedance method.

c. To study the resonance behavior in a series RLC circuit.

Theory

In this lab exercise, you will use the same hidden RLC circuit that you worked with last week and same set of apparatus. Make sure you have the same number on the box. You have already measured the values of R and R_L (the resistance of the inductor itself) but you may want to check those measurements again using the multimeter in resistance mode. You have also measured the values of L and C by measuring the time constants. Today you will measure L and C again by a different technique, called impedance method. In this lab, an AC signal (sinusoidal wave) from the function generator will be applied in the circuit for the measurements of current and voltage to determine the impedance.

When you have a RC series circuit in an AC source of angular frequency \( \omega \) (\( \omega = 2\pi f \)), you can calculate the capacitance (C) from the equation:

\[
C = \frac{V_{\text{max}, R}}{V_{\text{max}, C}} \cdot \frac{1}{\omega R}
\]

(1)

where \( V_{\text{max}, R} \) is the amplitude of the voltage across the resistor and \( V_{\text{max}, C} \) is the amplitude of the voltage across the capacitor. [You have to derive this equation in your report.]

Similarly, when you have a RL series circuit in an AC source of angular frequency \( \omega \) (\( \omega = 2\pi f \)), you can calculate the inductance (L) from the equation:

\[
(\omega L)^2 = \left( R \frac{V_{\text{max}, L}}{V_{\text{max}, R}} \right)^2 - R_L^2
\]

(2)

where \( V_{\text{max}, R} \) is the amplitude of the voltage across the resistor, \( V_{\text{max}, L} \) is the amplitude of the voltage across the inductor, and \( R_L \) is the resistance of the inductor and \( R \) is the external resistance connected in series to the inductor (that has resistance \( R_L \)). [You have to derive this equation as well in your report.]

You will also study the behavior of the series RLC circuit as a function of frequency and measure the resonance curve. The peak of the resonance curve occurs at a frequency given by the relationship

\[
f_0 = \frac{1}{2\pi \sqrt{LC}}
\]

(3)

Also, at resonance the current and voltage in the circuit are in phase with each other.
Apparatus

Same Circuit Board with hidden series RLC circuit used in last experiment, Decade Resistance Box, Function Generator, Digital Multimeter, Vernier LabQuest Interface, two Voltage Probes (one must be Differential Voltage Probe), a computer with Logger Pro software and connecting wires.

Description of Apparatus

You are going to use the same apparatus and set up that you worked last week. Make sure that you are working with the circuit box that has the same number as in last lab.

Procedure

Part I. Measurement of C

1. The computer and interface should already be turned on. Open Logger Pro software on the computer. Zero the sensors by connecting terminals of each of the probe.

Set up the Data collection sampling rate and duration \(\text{Experiment} \rightarrow \text{Data collection} \rightarrow \text{Sampling Rate} = 5000 \text{ and Duration} = 1 \text{ sec} \rightarrow \text{Done}\). Set the x-axis (time) ranging from 0 to 0.020 sec.

2. Connect the decade resistance box in series with the capacitor in your circuit (and nothing else!) and connect the function generator output across the combination of R and C as shown in Fig. 1. What is the impedance of the circuit when R and C are in series?

![Figure 1: Circuit diagram and V probes connections for measurement of C (and L).](image)

3. Set the decade resistance box to a resistance \(R_D = 200 \, \Omega\). Set the function generator to a sine wave output at a frequency of about 200 Hz and voltage amplitude \(V_P\) around 2 V. Note that at this frequency you will see approximately 4 complete cycles on the computer screen with a time scale going from 0 to 0.02 sec. The exact frequency is not crucial - you will measure whatever it is.

4. Connect voltage probe-1 across the capacitor and voltage probe-2 across the decade resistance box. **Probe-1 must be Differential Voltage Probe.** Make sure the other ends of the probes are connected to the corresponding channel (probe-1 into ch1 and probe-2 into ch2).
ch2) in the LabQuest interface device. Maintain the same polarity for both connections, i.e. measure the voltage from high to low (red to black) in the same direction for both as shown in Fig. 1.

5. Click on “collect” and observe the two sinusoidal waveforms in different colors one for voltage across the capacitor and another for voltage across the decade resistor. Click on “stop” to stop the data collection. Repeat collecting data again if the waveforms are not nice. **Observe the phase relationship between the two signals and comment on its significance in your lab report.**

6. Use the “automatic curve fitting” feature under Analyze in menu bar to fit both of the signals, sequentially, with a sine wave. Make sure you know which one you are fitting and make sure that the fit is a good one! The fitting program will return the values of the voltage amplitude (parameter A), the angular frequency \( \omega \) (parameter B) and the phases (parameter C) for each of the probes. Record them, keeping four significant figures in Table 1. The values of B, of course, should be very nearly the same (ideally, identical) for both of the signals but the amplitudes will be different.

7. Repeat the above measurements for a frequency of approximately 400 Hz. Print ONLY one graph from this part to include in your report.

### Part II. Measurement of L

1. For this part of the experiment, connect the decade resistance box in series with inductor in the circuit box (and nothing else!) and connect the function generator output across the combination of R and L as shown in Fig. 1. What is the impedance of the circuit when R and L are in series? Do not forget to include the resistance of inductor, \( R_L \)!

2. Follow the same procedure described above (steps 3 through 6). Record your results in Table 2.

3. Repeat the above measurements for a frequency of approximately 400 Hz. Print ONLY one graph from this part to include in your report.

### Part III. Measurement of the RLC resonance curve

1. Connect the function generator across your RLC series circuit such that all three elements are in the circuit. Do not connect the decade resistance box - leave it out of the circuit.

2. Connect voltage probe-1 across the entire circuit, i.e. across the output of the function generator. This will measure the voltage for the whole series circuit. Connect voltage probe-2 (Differential voltage probe) across the resistor in your circuit. This will measure the voltage across the resistor. Maintain the same polarity for both connections. **Note that voltage probe-2 is indirectly measuring the current in the circuit, since \( I = V/R \).**

3. Set frequency in the function generator to 100 Hz. Measure the voltage amplitudes for both signals from the voltage probes. As before, use the automatic fitting program to determine the voltage amplitudes (parameter A), the angular frequencies (parameter B), and the phases (parameter C) for each of the probes. Record your data in the Table 3.

4. Repeat the previous step to measure the voltage amplitudes for the following approximate frequencies:

\[ f = 100 \text{ Hz}, \ 200 \text{ Hz}, \ 250 \text{ Hz}, \ 300 \text{ Hz}, \ 350 \text{ Hz}, \ 400 \text{ Hz}, \ 500 \text{ Hz}, \ 600 \text{ Hz}, \ 800 \text{ Hz}. \]
Computation

1. Derive equations (1) and (2).
2. From the data in Table 1, calculate the capacitance using Eq. (1) for both frequencies.
3. Compare all the measurements of C obtained in this experiment as well as last week. Which of these do you think are most reliable? Do they agree within experimental errors? Calculate what you think is the best estimate for C, based on all your measurements. Calculate the phase difference from the fitting parameters and comment on the significance of the phase difference observed. Note: unit of phase in the fitting parameter is in radian.
4. From the data in Table 2, calculate the inductance using Eq. (2) for both frequencies.
5. Compare all the measurements of L obtained in this experiment as well as last week. Which of these do you think are most reliable? Do they agree within experimental errors? Calculate what you think is the best estimate for L, based on all your measurements. Calculate the phase difference from the fitting parameters and comment on the significance of the phase difference observed.
6. Make a graph of the ratio $V_{max,R} / V_{max}$ vs angular, $\omega$ as measured in part III from Table 3. What is the significance of this graph?
7. Determine the resonance frequency $f_0$, from the resonance curve and calculate LC using Eq. 3. How well does it agree with the value calculated from your best values for C and L?

Questions

1. Does the value of capacitance (C) (and the value of inductance (L)) depend on the frequency of the AC signal?
2. How does the current vary with changing frequency in both RC and RL circuits based on your results in part I and II? Why?
3. What happens to the phase relationship between V and $V_R$ (actually I) as the frequency passes through resonance? Why?
Data Sheet

Date experiment performed:

Name of the group members:

**Table 1. Measurement of C**

Resistance in decade box (R) =

<table>
<thead>
<tr>
<th>Set Frequency (f)</th>
<th>$V_{max,C}$ (Parameter A from Probe-1)</th>
<th>$V_{max,R}$ (Parameter A from Probe-2)</th>
<th>$\omega$ (Parameter B)</th>
<th>Phase difference ($\phi_C - \phi_R$) (Parameter (C₁ - C₂))</th>
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<td>200 Hz</td>
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<td>400 Hz</td>
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Capacitance (C) =

**Table 2. Measurement of L**

Resistance in decade box (R) =

Resistance of inductor (Rₜ) =

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<thead>
<tr>
<th>Set Frequency (f)</th>
<th>$V_{max,L}$ (Parameter A from Probe-1)</th>
<th>$V_{max,R}$ (Parameter A from Probe-2)</th>
<th>$\omega$ (Parameter B)</th>
<th>Phase difference ($\phi_L - \phi_R$) (Parameter (C₁ - C₂))</th>
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Inductance (L) =
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<th>Approx. Freq. (Hz)</th>
<th>( V_{\text{max}} )</th>
<th>( \omega )</th>
<th>( \phi_V )</th>
<th>( V_{\text{max},R} )</th>
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