Lab on the Series RL, RC and RLC Circuits and Resonance

**Purpose:**
1. To measure the value of a capacitor C and Inductor L using a series RC & RL circuits, in series with a sinusoidal voltage source.

2. To study the phase relationships between Voltage and Current for R, L and C.

3. To measure the resonance frequency of a series RLC circuit and compare with the theoretical value.

**Introduction**
This online lab exercise will allow you to simulate the time dependence of the current in a series RL circuit, a series RC circuit and finally a series RLC circuit, in each case driven by an alternating voltage source, using the online circuit-building tool at [www.falstad.com/circuit](http://www.falstad.com/circuit). You will be able to view the time dependence of the voltage and current (and their phase relationship) for all 3 components in the circuit.

The simulator also allows you to change the values of R (resistance), C (capacitance), and L (inductance) and observe resulting changes in the resonant frequency of the series RLC circuit.

**Software**
This lab runs in any web browser. Google Chrome is recommended. It can be run online or downloaded for offline viewing. The web address is: [www.falstad.com/circuit](http://www.falstad.com/circuit)

You can download an offline version as follows:

**Mac:** [http://falstad.com/circuit/CircuitJS1.dmg](http://falstad.com/circuit/CircuitJS1.dmg)

**MS Windows:** [http://falstad.com/circuit/circuitjs1.zip](http://falstad.com/circuit/circuitjs1.zip)

Downloading the program allows it to work faster and you may encounter fewer delays caused by network speed. If you do not want to download it, you can work online on the website. Whether running online or offline, the actual procedure of conducting the simulation and analysis is the same.

**Running the experiment** *(the data sheet is on page 6)*
Open the program: circuitjs1.exe (Or circuitjs1.dmg for mac, Or use the online version).

**Part 1, Measuring C in a series RC circuit**

1) Click ‘Circuits’ in the top menu and click ‘A/C circuits’, then select ‘Capacitor’.

2) Let’s set the capacitor value, \( C = 2 \, \mu \text{F} \). To do this, move the mouse over the capacitor till its color becomes light blue, then right click it and edit and put the value as 2 micro Farads, i.e. type 2u (and it already knows that the unit is F:Farad) and click ‘apply’ then click ‘OK’.

Now, let’s set the resistance value, \( R = 200 \, \Omega \). To do this, move the mouse over the Resistor till its color becomes light blue, then right click it and edit and put the value as 200 (and it already knows that the unit is \( \Omega \)) and click apply then click OK.
Finally, let’s set up an AC voltage source with amplitude 1 Volt and frequency 200 Hz, as follows: Move the mouse over the Function Generator (the A/C voltage source), till its color becomes light blue, then right click it and change the value of Max Voltage to 1 (and it already knows that the unit is v: Volts). Keep the type of the wave form as is (A/C), and put the value of the frequency as 200 (and it already knows that the unit is Hz: Hertz). Keep the Phase offset (degrees) as is (the default of zero), and then click ‘apply’ then click ‘OK’.

3) Let’s set up the way in which the waveforms are displayed at the bottom of the screen (the “Dock). In the graph area, at the bottom, click the gear icon (or right click in that area), and in ‘Horizontal scale’, set the scroll speed to 2 ms/div (using the arrow), then click ‘apply’ and then ‘OK’.

4) To view the properties of the resistor, voltage source and capacitor:
   • move the mouse over the resistor until its color becomes light blue, then right click it and edit and select view in scope.
   • Do the same for the function generator (the A/C voltage source).
   • The scope for the capacitor voltage is already available at the bottom (1st graph on left of the Dock). If you have deleted by accident, view it in the scope as well by doing the same procedure as for the resistor and function generator.

Make the Simulation speed medium, in the upper right corner, then click Run/Stop, and after about 4 cycles, click the Run/Stop again to stop the running.

5) Move the mouse to:
   • the graph of the capacitor output in order to find the value of the voltage (green curve) at a peak (say the second peak), and also record the time at that peak, \( t_{V,C} \). Note that if you want to increase the number of decimal places in the time reading you can decrease the scroll speed per division in the properties of each graph (this makes the time scale more sensitive).
   • the resistor graph to do the same thing, i.e. find the peak value of the voltage (green curve) across the resistor, and the time at that peak, \( t_{V,R} \). Make sure that you examine the same peak for the capacitor and the resistor. In other words, if you are looking the 2nd set of peaks for the capacitor, you must also look at the 2nd peak for the resistor. This is so that you can compare times within the same cycle.

We expect that \( C = \frac{V_{R,\text{max}}}{V_{C,\text{max}}} \times \frac{1}{\omega R} \) where \( \omega = 2\pi f \), \( f \) is the frequency, in this procedure \( f = 200 \text{ Hz} \).

From the above formula, calculate the “observed” value of capacitance, \( C \) and compare it with the capacitance that you set, which was \( 2 \mu F \).

Figure 1: Example R\text{C} circuit (top) with an AC voltage source. Graphs of the voltage across, and current through, the three components in the circuit are shown in the “Dock” below the circuit diagram.
6) Calculate the difference in the times you recorded in step 5 above for the peaks of $V_R$ and $V_C$:

$$\Delta t = t_{V_R} - t_{V_C}$$

Using this time, find the phase difference (phase angle) between $V_R$ and $V_C$ by using the equation:

$$\Delta \phi = \omega \Delta t = 2\pi f \Delta t, \quad \text{& compare the value of } \Delta \phi \text{ to the value of } \pi/2. \text{ Are they approximately equal?}$$

You should find that, for the resistor the voltage curve and the current curve are in phase (i.e. 0 phase difference) and so you cannot “see” the voltage and current curves separately, as they overlap perfectly. But $I_R = V_R/R$, so $I_R$ should be of different magnitude to $V_R$. The simulator plots $V_R$ and $I_R$ on top of each other which is probably a problem that sometimes occurs with the simulator. It is true that they are in phase, but they have different values. If you want to see $I_R$, right click the graph of $R$ and select properties and uncheck show voltage and leave show current checked.

7) Now repeat the above steps with a different AC power source frequency: 400 Hz.

**Part 2, Measuring L in a series RL circuit**

1) You will now repeat the procedures of part 1, but with an inductor. You can delete the capacitor (right click & delete) and replace it by an inductor: click ‘Draw’ in the top menu and select ‘Passive components’ and select ‘Add inductor’. Place it in the place of the capacitor. Right click the Source and select view it in scope. You may prefer to use an alternate way (choose the way you prefer) to build the circuit as from the beginning: Open the program, Click ‘Circuits’ and click ‘A/C circuits’ and select ‘inductor’ (instead of capacitor). Set up a 0.1 Henry inductor by changing the value of the inductor to 0.1 (and it already knows that it is in H: Henry). The resistor can be 200 Ohms, as before. The power source should also have the same maximum voltage (1 Volt) & frequency (200 Hz) as before. As in step 3 of part 1, set the scroll speed to 2 ms/div. Also, view resistor, $R$ (200 $\Omega$) in scope.

As we saw in exp. 8, a real coil would have internal resistance, $r_L$. Click ‘Draw’ and choose ‘Select/Drag Sel’ and the drag the lower terminal of $L$ to make room for inserting, $r_L$. Click ‘Draw’ and ‘Add Resistor’ and insert $r_L$ as shown in Fig. 2. Edit the value of $r_L$ to 20 (unit already known as ohms). Add wires as shown to prepare for adding a ‘voltmeter/scope’. Click ‘Draw’ and select ‘Outputs and Labels’ and choose ‘Add voltmeter/ scope probe’. Note polarity. Move the mouse to the voltmeter until its color becomes light blue, then right click it and select ‘View in new Scope’. This will add a 3rd graph (on the far right) for the voltage and current of the real coil ($L$ combined with $r_L$). **Note that in a real lab circuit**, we cannot measure the voltage across the ideal inductor, $L$, (the first graph on the left), since, as we mentioned in exp. 8, we cannot physically isolate $L$ from $r_L$. Add a box as shown in fig. 2, around $L$ and $r_L$ from ‘Draw’ and ‘Outputs and labels’. Finally, set the Simulation speed to medium.

Click ‘Run/Stop’ to simulate, and click ‘Run/Stop’ again after about 3 or 4 cycles. Observe the maximum in the voltage across the inductor, $V_{L+r_L,max}$ (3rd graph) and, again, the maximum in the voltage across the resistor, $V_{R,max}$.

To “find” $L$, we can use the theoretical equation $(\omega L)^2 = (R \times \frac{V_{L+r_L,max}}{V_{R,max}})^2 - r_L^2$. Compare your finding with the value that you set (0.1 H). **Derive this eqn.** Hint: Using ohm’s law find $V_{L+r_L,max}$ & $V_{R,max}$, divide & cancel the current then solve for $(\omega L)^2$. 

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2) As in Part 1, find the phase angle (phase difference), \( \Delta \phi \) between the voltage of the real coil (\( L \) combined with \( r_L \)) and the voltage of the resistor by noting the time at which corresponding peaks in each signal occur. This time is,

\[
\Delta t = t_{V,R} - t_{V,L+r_L}.
\]

Should we expect the value of \( \Delta \phi \) to be \( \frac{\pi}{2} \) or \( 0 < \Delta \phi < \frac{\pi}{2} \)? Why?

Again, make sure to examine the same peaks in the “scope” output for an accurate comparison of the time within the same cycle. Which voltage is leading? Is \( V_{L+r_L} \) leading \( V_R \) or is \( V_R \) leading \( V_{L+r_L} \)?

Using the graph of \( V_L \) (1st graph on the left) & \( V_R \) (middle graph) find \( \Delta \phi \) between \( V_L \) & \( V_R \). Compare the value of \( \Delta \phi \) to \( \pi/2 \). Is it approximately equal? Which is leading, \( V_L \) or \( V_R \)? (Note that in a real lab we cannot measure \( V_L \) as isolated from \( r_L \)).

3) Now repeat steps 1) and 2) with a different AC power source frequency: 400 Hz. Should the value of \( L \) change?

**Part 3, Resonance in a series RLC circuit**

1) Click **Circuits** in the top menu, then **A/C** then select **Capacitor**. Set the values as shown in Figure 3. The max (peak) voltage for \( V_{\text{source}} \) should now be 5 Volts.

We are going to delete the bottom wire and replace it by an inductor \( L \). To do that, move the mouse to the bottom wire and right click and select **delete**. Then, click ‘**Draw**’ in the top menu and select ‘**Passive components**’ and select ‘**Add inductor**’. Here we are going to assume an ideal inductor, \( L \), assuming we combined the internal resistor \( r_L \) of the real inductor with the \( R \) resistor for analysis purposes. Place it in the place of the wire that we deleted by clicking and dragging from the bottom end of the capacitor to the bottom end of the voltage source. **Make sure that the inductor windings are as shown in Figure 4; if not then right click the inductor and select swap terminals.** Edit its value to 0.1 (it knows the unit: H).

So as not to keep adding more inductors, in **Draw** select the last item **Select/Drag Sel.** Now the mouse is back to normal.

2) Click the settings of the graph in the bottom Dock and change the **Scroll speed** to 2ms/ div. Move the mouse to the inductor until its color becomes light blue, and right click it and select **view in scope**, and do the same for the resistor. This is to show the voltage wave form across the inductor (green curve) and also the current wave form in the inductor (yellow curve) and voltage and current for \( R \). **What is the relation of this current in \( R \) to the current in \( L \) and in \( C \) in this circuit? Why?**

3) Run the simulator by clicking **Run/Stop**. Keep running the simulation until the curves become smooth. You can increase the speed of the simulation using **Simulation speed** at the top right of the window. Stop the simulation by clicking **Run/Stop**.
Measurement

- Find the peak value of the current (yellow curve) by placing the mouse on the peak of the yellow current curve and reading the value of the current displayed by the simulator at the peak, where you have placed the mouse. (Remember: the current through the circuit is the same for all 3 devices since they are in series!)
- Repeat this measurement of peak current with the frequency of the source changed to 300 Hz. Each time, record the peak (Maximum) of the current curve. You need to run, stop and then re-run for each separate measurement.
- Then repeat for f= 356 Hz, f= 400 Hz and 500 Hz. Finally try at f= 350 Hz and then 360 Hz. Use the table below to log the variation in peak current, $I_{\text{max}}$.

Analysis

(a) Plot a graph of the peak value of the current versus frequency. You can add more values of the frequency if you want to, by taking more measurements as above. Estimate the “resonance” frequency from your graph – the frequency at which the peak current is maximum – and compare with the calculated resonance frequency for this circuit using

$$X_{\text{L at res}} = X_{\text{C at res}} \rightarrow f_{\text{resonance}} = \frac{1}{2\pi \sqrt{LC}}.$$  

For such a RLC series circuit, why does the peak of the current have its maximum value at the resonance frequency? And why is $X_{\text{L at res}} = X_{\text{C at res}}$? Let’s explore that question. First, notice that in this series RLC circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}, \text{ where } X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}, \text{ } \omega = 2\pi f.$$  

From this relation, Z is always greater than or equal to R. Moreover, at low frequency $X_C$ is large, and $X_L$ is low. As frequency is increased, $X_C$ decreases, and $X_L$ increases. So, there is a value of frequency where $X_C$ becomes equal to $X_L$. Then $Z = R = Z_{\text{minimum}}$, which is the minimum value that Z can take in this circuit. This frequency is called the resonance frequency in this circuit. And since $I_{\text{max}} = \frac{V_{\text{max \source}}}{Z}$, then $I_{\text{max}}$ has its largest value at $f_{\text{resonance}}$.

(b) For any frequency (say 100 Hz), find the phase difference angle between the voltage across L ($V_L$), and the voltage across C ($V_C$), by recording the times of corresponding peaks (same peaks, say the first peak after the vertical line of the y axis in both graphs) using the mouse and then using $\Delta \phi = \omega \Delta t = 2\pi f \Delta t$. As in the rest of this lab, you must use corresponding peaks to be able to find corresponding phase difference. To emphasize, the “corresponding peaks” means: if you use the first positive peak of $V_C$ after the vertical line of the y axis you must also use the first positive peak of the $V_L$ after the vertical line of the y axis to record the correct corresponding times to find $\Delta t$.

You can run and stop the simulator until you are able to see simultaneously displayed in the scopes the same corresponding peaks for $V_C$ and $V_L$ voltages (green curves for C and L), as in the example shown in Figure 4. Which is leading: $V_L$ or $V_C$?

Note that if $\Delta t= 0 \text{ s}$, then something is wrong. Review step 1 in part 3 concerning the windings of the inductor.

(c) What is the phase difference between the $I_{\text{R}}$, the current through the resistor and the $V_{\text{R}}$, the voltage across R?
DATA TABLES

Part 1: RC circuit

<table>
<thead>
<tr>
<th>Source frequency: 200 Hz</th>
<th>Source frequency: 400 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{V,R}$:</td>
<td>$t_{V,R}$:</td>
</tr>
<tr>
<td>$t_{V,C}$:</td>
<td>$t_{V,C}$:</td>
</tr>
<tr>
<td>$\Delta t = t_{V,R} - t_{V,C}$:</td>
<td>$\Delta t = t_{V,R} - t_{V,C}$:</td>
</tr>
<tr>
<td>Phase difference (R – C):</td>
<td>Phase difference (R – C):</td>
</tr>
<tr>
<td>$\Delta \phi =$</td>
<td>$\Delta \phi =$</td>
</tr>
<tr>
<td>$\Delta \phi$ in terms of $\pi/2$:</td>
<td>$\Delta \phi$ in terms of $\pi/2$:</td>
</tr>
<tr>
<td>Measured $C = \frac{V_{R,\text{max}}}{V_{C,\text{max}}} \times \frac{1}{\omega R}$:</td>
<td>Measured $C = \frac{V_{R,\text{max}}}{V_{C,\text{max}}} \times \frac{1}{\omega R}$:</td>
</tr>
</tbody>
</table>

Part 2: RL circuit

Derive the eqn. that we use to find $L$:

<table>
<thead>
<tr>
<th>Source frequency: 200 Hz</th>
<th>Source frequency: 400 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{V,R}$:</td>
<td>$t_{V,R}$:</td>
</tr>
<tr>
<td>$t_{V,L}$:</td>
<td>$t_{V,L}$:</td>
</tr>
<tr>
<td>$\Delta t = t_{V,R} - t_{V,L}$:</td>
<td>$\Delta t = t_{V,R} - t_{V,L}$:</td>
</tr>
<tr>
<td>Phase difference ($R - L + r_L$):</td>
<td>Phase difference ($R - L + r_L$):</td>
</tr>
<tr>
<td>$\Delta \phi =$</td>
<td>$\Delta \phi =$</td>
</tr>
<tr>
<td>$\Delta \phi$ in terms of $\pi$:</td>
<td>$\Delta \phi$ in terms of $\pi$:</td>
</tr>
<tr>
<td>Find measured $L$ using: $\left(\omega L\right)^2 = \left(\frac{V_{L,\text{max}}}{V_{R,\text{max}}}\right)^2 - r_L^2$</td>
<td>Find measured $L$ using: $\left(\omega L\right)^2 = \left(\frac{V_{L,\text{max}}}{V_{R,\text{max}}}\right)^2 - r_L^2$</td>
</tr>
</tbody>
</table>

find $\Delta \phi$ between $V_L$ and $V_R$; $\Delta \phi =$ Compare the value of $\Delta \phi$ to $\pi/2$: $\Delta \phi =$ Which is leading, $V_L$ or $V_R$? ....

Should the value of $L$ change for different frequencies? ........

Part 3: RLC circuit

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>Peak of current</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
</tr>
<tr>
<td>350</td>
<td></td>
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<td>356</td>
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<td>360</td>
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<tr>
<td>400</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

$f_{\text{resonance}}$ from graph = ............... Hz

$f_{\text{resonance}}$ from calculation = ............... Hz

Show your work for calculation of $f_{\text{resonance}}$:

You can copy this table and values into your lab report along with all the other requirements of the lab report: the graphs of all parts, calculations,.....etc.