RC Circuits

Purpose - To study the properties of an RC circuit. To measure its time constant. To use the time constant to find values of some unknown capacitors. To confirm the series and parallel formulas for capacitors.

Apparatus - Computer, printer, ULI interface with 2 voltage probes, 5V power supply, decade resistance box, double-throw switch, two capacitors, wires.

References: Serway and Beichner, Physics for Scientists and Engineers (5th ed.), Chap. 28, sec. 28.4, pp. 882 - 887; Cutnell and Johnson, Physics (5th ed.), Chap. 20, secs. 20.12 and 20.13, pp. 611 - 613.

CIRCUIT DIAGRAM

Theory:

When the capacitor is being charged through a resistance R, the voltages around the loop satisfy:

\[ \varepsilon - V_C - V_R = 0 \]

where \( \varepsilon \) is the emf (voltage) of the battery eliminator and 

\[ V_C = \frac{q}{C} \]

is the voltage drop across the capacitor (\( \Delta V_C \))

\[ V_R = I R \]

is the voltage drop across the resistor (\( \Delta V_R \))

The time-dependent (transient) behavior of \( V_C \) and \( V_R \) is given by the following equations, assuming that the capacitor is initially uncharged:
\[ V_C(t) = \mathcal{E} \left[ 1 - \exp\left( -\frac{t}{\tau} \right) \right] \quad \text{where} \quad \tau \quad \text{is the “time constant” of the circuit} \]

\[ V_R(t) = \mathcal{E} \exp\left( -\frac{t}{\tau} \right) \quad \tau = RC \]

The charge on the capacitor when it is fully charged is: \[ Q = CE \]

When the capacitor is being discharged through a resistance \( R \), the time-dependent (transient) behavior of \( V_C \) and \( V_R \) is given by:

\[ V_C(t) = V_o \exp\left( -\frac{t}{\tau} \right) \quad \text{where} \quad V_o = Q/C \]

\[ V_R(t) = -V_o \exp\left( -\frac{t}{\tau} \right) \]

**Procedure:**

1. Wire the circuit as shown. Note that the capacitor has a polarity - connect it properly! Use the 5V output of the power supply. Connect Probe #1 across the capacitor and connect Probe #2 across the resistor, using the proper polarities.

2. Make sure that the capacitor is initially discharged by placing the switch in the discharge position, with \( R \) set to zero. Then set the resistance, using the decade box, to a suitable value (compute an approximate value of \( R \) to give \( \tau \sim 5 \) sec.). Click on “Collect” on the computer screen and then, shortly after the data starts collecting, throw the switch to the CHARGE position. Wait until the capacitor is fully charged and throw the switch back to the DISCHARGE position. You will now have, in one graph, both the charging and discharging curves for the capacitor and the resistor. **Save the data** in a temporary file before continuing!

3. For both the charging and discharging portions of the curves for the capacitor, you will select an appropriate region and fit the data with the appropriate function to determine the time constant for the circuit. Your instructor will show you how to do this using the computer software. The region you select should begin and end on the portion of the curve that is changing significantly. After marking the region, click on “Analyze” and select “Curve Fit”, then choose the function (“Inverse Exponent” for the charging portion of the curve, “Natural Exponent” for the discharging portion of the curve) Then click on “Try Fit” and finally click on “OK” to show the fitting
parameters in a box on the graph. **Note that \( \tau \) is the reciprocal of the fitting constant \( C \) (this \( C \) is not the capacitance!). If the two values of \( C \) are in good agreement and if the “Root MSE” for each is less than ~ 0.01, you should record both values of \( \tau \) obtained by this procedure. If the values are not in good agreement, you should repeat the analysis, or the entire measurement if necessary, until satisfactory agreement is achieved. You will use the average of these two values of \( \tau \) in your subsequent analysis of the data. You should also estimate the experimental uncertainty in the average value of \( \tau \).

4. For this first measurement, examine the table of values for \( V_C(t) \) and \( V_R(t) \) and, for five different values of \( t \) spanning the time during which (i) the capacitor is charging and (ii) the capacitor is discharging, compute the value of \( (V_C(t)+V_R(t)) \) and record these values. The table can be displayed by clicking on “New Tall Window - Table” and you can scroll down the (very long) list of values.

[Physics 2.5 only] Using the “Integrate” function in the analysis software, integrate \( V_R(t) \) separately for both the charging process and the discharging process. Record the values of these integrals and also record the voltage \( V_C \) on the fully charged capacitor.

5. Repeat steps 2 and 3 (but not 4) above for at least three other significantly different values of \( R \). These should span the entire range for which you can measure the time constants, i.e. from \( \tau \sim 0.1 \) sec to \( \tau \sim 10 \) sec. You will now have at least four values of \( \tau \), corresponding to four different values of \( R \), which can be used to construct a graph of \( \tau \) vs. \( R \).

6. Repeat the procedure described in steps 2, 3, and 5 (omitting 4) for the other capacitor, thereby obtaining at least four values of \( \tau \) as a function of \( R \) for the second capacitor.

7. Connect the two capacitors in series and measure the time constant using the method described in 2 and 3 above, for one value of \( R \) only.

8. Connect the two capacitors in parallel and measure the time constant using the method described in 2 and 3 above, for one value of \( R \) only.

Your **Tabulated Results** should contain (see sample tables at end):

A. The experimental values of \( \tau \) for each of the individual capacitors and for the series and parallel combinations.
B. The values of C for each of the two capacitors, determined from the graphs of $\tau$ vs. R.

C. The experimental and theoretical capacitances for the series and parallel combinations. The experimental values are determined from the measured values of $\tau$, while the theoretical values are computed using the appropriate formulas for series and parallel combinations of capacitors and the experimentally determined values of C for the individual capacitors.

D. [Physics 2.5 only] The values of Q determined by (i) $Q = I \int dt$ and (ii) $Q = C V$ for both the charging and discharging processes in part 4).

**Graphs:** Construct the two graphs (one for each capacitor) of $\tau$ vs R.

Questions to be answered in your report:

1. Does your graph of $\tau$ vs. R pass through the origin? Should the graph necessarily pass through the origin? What does a non-zero intercept signify?

2. Discuss your observations of $[V_C(t) + V_R(t)]$ for both the charging and discharging processes (part 4). [Physics 2.5 only] Compare and discuss the values of Q determined by the two different methods in part 4).

3. Do your experimental values for the series and parallel combinations of capacitors agree with the theoretically calculated values? Discuss in terms of the experimental errors involved.

4. Your capacitors were charged to a maximum voltage of 5 volts. In each of the four cases (that is for the individual capacitors and for the series and parallel combinations), when the capacitors were fully charged, what was:
   A. The charge (in microcoulombs) stored on the plates of each capacitor.
   B. The energy (in microjoules) stored in each capacitor.
Table I:

<table>
<thead>
<tr>
<th>Capacitor, C</th>
<th>Resistance, R(Ω)</th>
<th>Charging, τ (s)</th>
<th>Discharging, τ (s)</th>
<th>Average τ (s)</th>
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<tbody>
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Table II:  (Experimental values must be accompanied by estimated uncertainty)

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<tr>
<th>Experiment</th>
<th>C₁</th>
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<th>C_{eq} - Parallel</th>
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<tr>
<td>Theory</td>
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